Wrocław university of science and technology
Faculty of Microsystem Electronics and Photonics

Designing, manufacturing and testing capstan drive for quadruped robots

Author: Jakub Tomaszewski
Class: Intermediate Project
Supervisor: Ph.D. Witold Paluszyński
Date: January 26, 2022

Abstract

The highly dynamic legged jumping motion has been one of the greatest challenges in robotics oriented research. The project proposed a nonlinear optimization that generates an optimal trajectory in the launch phase that minimizes the joint torques and accelerations. This method allows to generate the optimal trajectory of the leg movement during the vertical jump and to select all necessary parameters of the leg to perform the jump.

A series of optimizations for different parameters was performed. Based on this, a capstan drive was designed, manufactured, and tested that could be used as an actuator of highly dynamic legs for quadruped robots.

1 Introduction

The purpose of this project is to design, manufacture and test a capstan drive [5], [1] for a quadruped robot. The main assumption of the capstan drive is to use it in a robotic leg that will be able to perform a vertical jump. The actuator will consist of a brushless motor, capstan, cable, and tensioning system. The mechanism should provide zero backslash, low inertia, and backdrivability, which are all required in a quadruped robot. All parts of the capstan drive will be 3D printed (expected bearings, screws, and rope). To determine the requirements for the capstan drive and the entire leg, a nonlinear optimization was performed which generates the optimal trajectory of the leg movement during the vertical jump [2], [7].

1.1 Assumptions

The main assumptions of the project include:

- Determine the assumption about the jump
- Defining the objective function of the optimization
- Defining the necessary constraints n of the optimization
- Develop an optimization which generates the leg trajectory during the jump
- Performing a series of simulations for various parameters
- Verification of the obtained results with reality
- Determine the parameters of the leg
- Determine the parameters of the actuators
- Designing a capstan drive
- 3D print a prototype of the capstan drive
- Manufacture a capstan drive
- Implementing a program to control the capstan drive
- Assembly a capstan drive and performing tests

2 Trajectory optimization in the launch phase

The purpose of this task was to obtain the requirements of the parameters that the leg should have to perform a vertical jump. The optimization task was also aimed at understanding the forces, the relationship between the parameters, and the physical laws related to the execution of the jump.

During the optimization, a leg model with two degrees of freedom was used. It consist of the first joint, which is the hip (corresponding to a angle q_1), and the knee (corresponding to a angle q_2). This model was used to estimate the torque, speed, and leg dimension requirements to perform jumping motion. It is assumed that both links in the leg model have the same length l. The motors and gearboxes of both joints are placed in the base, which is the center of mass of the leg (CoM). In the optimization, it was assumed that both links are massless, so that all mass is lumped at the base, which is mounted on a vertical rail. The important fact is that while the foot of the leg and the base are vertically aligned, then only the knee joint is actuated to perform a vertical jump while the hip joint is passive. A horizontal offset between the foot and the base has been added in order that both joints have an influence on the production of force when jumping. It is assumed that the horizontal displacement of the base will be denoted as x, while the vertical will be denoted as z.

2.1 Cost Function of nonlinear optimization

In a brushless motor, the torque is proportional to the current of the motor and the power is proportional to the square of the current. When performing a movement, it is important to minimize the energy required for its execution. Due to the physical limitation of the impetuous changes in acceleration of the joint, it is prudent to minimize the acceleration during the movement. That is why it is intuitive to minimize torque and acceleration [2]. To reduce sudden changes that can cause mechanical damage and high energy consumption, the changes in torque and acceleration are also minimized. To make the optimization a finite dimensional problem, the time trajectories were sampled at N grid points. The cost function of the optimization task in the launch phase is formulated as follows:

$$J = \sum_{k=1}^{N}[|\tau_k^s| + |\ddot{q}_k^s| + |\tau_k^s - \tau_{k-1}^s| + |\ddot{q}_k^s - \ddot{q}_{k-1}^s|] \quad (k = 1, 2, ..., N) \tag{1}$$

where τ_k^s and $\ddot{q}_k^s$ are the joint torque and acceleration at grid point k, respectively; s is the number of a joint and it is equal 1 for the hip and 2 for the knee.

The vector of optimization variables x_{opt} is defined as:

$$x_{opt} = [\alpha_{F1}, \alpha_{F2}, \alpha_{F3}, \alpha_{F4}, \alpha_{F5}, T_{st}, vec(q), vec(\dot{q})], \tag{2}$$

where $\alpha_{F(1-4)}$ are the Bèzir polynomial coefficients of the ground reaction force, $vec(q), vec(\dot{q}) \in R^{2xN}$, are the vectors of joint angles and velocities at each grid point, and T_st is the duration of launch phase.

2.2 Constraints of the nonlinear optimization

The dynamics of the leg model was defined as a single point mass accelerated due to the ground reaction force (GRF), which is the only external force that could change the mechanical energy of the system [7]. The equation of motion is the following:

$$\ddot{z} = \frac{F_z}{m} - g, \tag{3}$$

where F_z is the vertical component of the GRF, m is the lumped mass at the base, and g is the gravitational acceleration. A 5^{th} order Bèzier polynomial was used to parametric GRF curve, and its coefficient are $\alpha_F := [\alpha_{F1}, \alpha_{F2}, \alpha_{F3}, \alpha_{F4}, \alpha_{F5}, 0]$. The last coefficient was set as 0 to ensure a smooth and feasible ground reaction profile [6].

In order to set all necessary constraints in optimization, a velocity and position of the base are needed. The Bèzier polynomial is the linear combination of a Bernstein polynomial basis and it is defined as:

$$C_M(s) = \sum_{i=0}^{M} \alpha_i B_{i,M}(s), \tag{4}$$

where M is the order of Bèzier polynomial, α_i is the coefficient for the i^{th} Bernstein polynomial $B_{i,M}$.

The GRF can be integrated analytically [3] providing a 6^{th} order Bèzier polynomial with coefficient $\alpha_{\dot{z}}$, which describes the base velocity curve. The linear relationship for integrating the GRF Bèzier coefficient to get the velocity Bèzier coefficient can represent as:

$$\frac{M+1}{T_{st}}\theta(M, T_{st})\alpha_{\dot{z}} = [\frac{1}{m}\alpha_F^T - g, \dot{z}_0]^T, \tag{5}$$

where T_{st} is the duration of launch phase, $\dot{z}_0$ is the initial velocity and $\theta \in R^{(M+2)x(M+2)}$ is a matrix which is defined as:

$$\theta_{i,j} = \begin{cases} -1 & \text{for} \quad j = i = 1, 2, ..., M+1 \\ 1 & \text{for} \quad j = i+1 = 2, 3, ..., M+2 \\ \frac{T_{st}}{M+1} & \text{for} \quad i = M+2, j = 1 \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

Similarly, given an initial position z_o, a coefficient α_z could be obtained after integrating the velocity curve.

2.2.1 Constraints of joint positions

It is important to satisfy the forward kinematics of the leg. To ensure that the foot position at each grid point is consistent with the base position resulting from the ground reaction force, the following constraint has been added:

$$rot(q_k^1) \begin{bmatrix} l \\ 0 \end{bmatrix} + rot(q_k^1 + q_k^2) \begin{bmatrix} l \\ 0 \end{bmatrix} + \begin{bmatrix} -\Delta x \\ z_k \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (k = 1, 2, ..., N) \tag{7}$$

where $rot \in SO(2)$ is the rotation matrix, z_k is the position of the base at each grid point; $q_k = [q_k^1, q_k^2]^T$ are the joint angles of hip and knee at each grid point, l is the length of links and Δx is the added horizontal offset between the foot and the base.

2.2.2 Constraints of joint velocities

Similarly as the position to ensure that the foot velocity at each grid point is consistent with the base velocity resulting from the ground reaction force, the following constraint has been added:

$$J(q_k)\dot{q}_k + \begin{bmatrix} 0 \\ \dot{z}_k \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (k = 1, 2, ..., N) \tag{8}$$

where $J(q_k)$ is the manipulator Jacobian of the foot relative to the hip, $\dot{q}_k = [\dot{q}_k^1, \dot{q}_k^2]^T$ are the joint velocities of hip and knee at each grid point and $\dot{z}_k$ is the velocity of the base at each grid point.

2.2.3 Limits of the joint torques

Since both links are assumed to be massless, joint torque $\tau_k \in R^{2x1}$ could be reconstructed using:

$$\tau_k = J(q_k)^T F_{GRF,k} \quad (k = 1, 2, ..., N) \tag{9}$$

Then, regarding to the capacity of the motor and the gearbox, it is important to limit the output torque of the leg's joints. Therefore, the following torque constraints are introduced:

$$\tau_{lb} \leq \tau_k \leq \tau_{ub} \quad (k = 1, 2, ..., N) \tag{10}$$

where τ_{lb} is the lower bound of the torque, and τ_{ub} is the upper bound of the torque.

2.2.4 Limits of the joint angles and velocities

It is important to set the feasible limit of joint angles and velocities that can be reached with each joint. The joint angle limits determine the range of leg movement, while the joint velocities depend on the selected motor, a gear ratio, and a selected power supply, as well as the weight and inertia of the robot. With that in mind, the following joint constraints should be set as follows:

$$\begin{aligned} q_{lb} \leq q_k \leq q_{ub} \\ \dot{q}_{lb} \leq \ddot{q}_k \leq \dot{q}_{ub} \quad (k = 1, 2, ..., N) \end{aligned} \tag{11}$$

where lb and ub subscript denotes the lower and upper bound, respectively.

2.2.5 Initial and final posture of the robot

The optimization problem generates a trajectory between the initial and final posture of the leg. Therefore, the initial and final configuration of the leg should be added as constraints.

$$\begin{cases} q_k = q_{k,d} \\ \dot{q}_k = \dot{q}_{k,d} \quad (k = 1, N) \end{cases} \tag{12}$$

where $q_{k,d}$ and $\dot{q}_{k,d}$ are given joint angle and velocity at time step 1 and N (moment of take off), respectively

2.3 Results

Having defined the objective functions and all constraints, the next step is to implement the nonlinear optimization that will generate the trajectory of the leg. All necessary functions and the entire program was developed in Matlab. It is a paid program, but it can be used for one month for free. In this case, an educational license was used which provides access to the programming environment with all available toolboxes. Matlab's function `fmincon` was used to search local optimal solutions. As a result of the optimization task, a vector of optimal parameters is obtained, which the solver finds based on a given objective function and defined constraints. Based on the results, many important variables can be obtained, which after verification, can be used in the control algorithm to perform the desired vertical jump. The theoretical height of the jump can be calculated as:

$$h_{jump} = h_{t0} + \frac{v_{t0}}{2g} \tag{13}$$

where h_{t0} and v_{t0} are velocity and height at the moment of take off (in the last moment of simulation), so the value at time step N.

2.3.1 Reduction 6:1

To perform the optimization, the implemented functions and constraints require many parameters as an input. One of the most important is the gear ratio of the actuators, because it affects the maximum values of the speed and torque. Therefore, the gear ratio selection is a trade-off between speed and torque, with are the most important parameters. Another important parameter is the length of the links, which affect the maximal vertical speed of the leg's base and the produced torque in each joint. The last important parameter is the approximate mass of the leg with respect to which the simulation will be performed. During the optimization, it was assumed that, regardless of the selection of other parameters, a mj5208 brushless motor will be used (with the peak torque equal 1.7 Nm). In the first optimization, the following parameters were chosen:

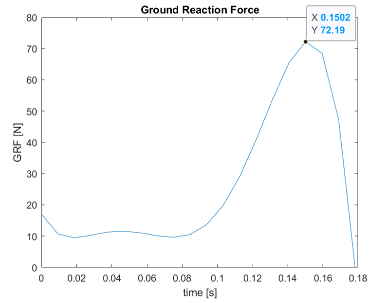
- gear ratio: 6:1
- links length: 0.11 m
- mass of the leg: 1,0 kg

Based on the parameters and the knowledge about parameters of the chosen motor, and assumption that the capstan drive has efficiently of 80% the following parameters can be calculated:

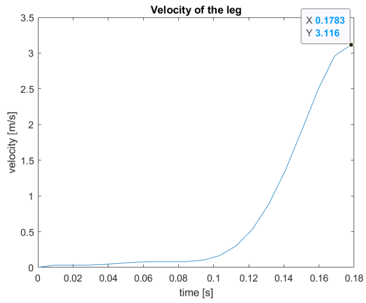
- maximum output speed of the gearbox: $138.23\frac{rad}{s}$
- maximum output torque of the gearbox: 8.16 Nm

The defined optimization's constraints require the initial and final position and velocity of the leg's foot. The position was set in Cartesian coordinated and then using the inverse kinematic of the leg, the angles of each joint were calculated. Taking into account the physical limitation of the leg mechanism, the chosen motor, and the defined gear ratio, the following values have been set:

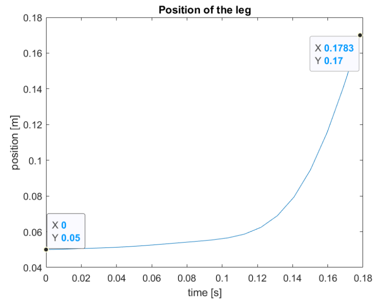
- initial value in step 0:
  - position: x = 0.03 m, z = -0.05 m
  - velocity: $0 \frac{rad}{s}$
- final value in step N:
  - position: x = 0.03 m, z = -0.17 m
  - velocity: $45 \frac{rad}{s}$



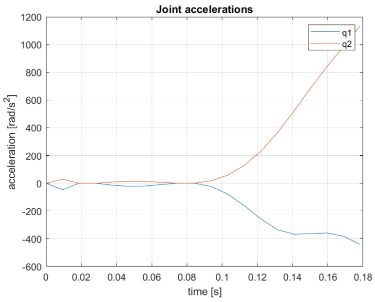
(a) Ground Reaction Force



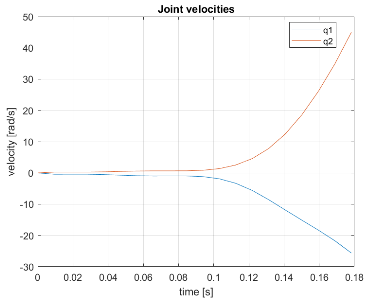
(b) Velocity of the leg base



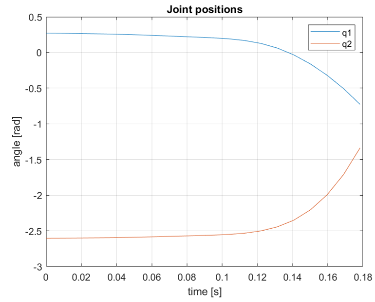
(c) Position of the leg base



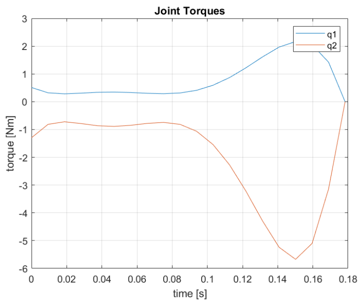
(d) Acceleration in leg joints



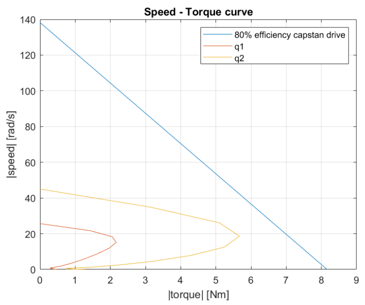
(e) Velocities in leg joints



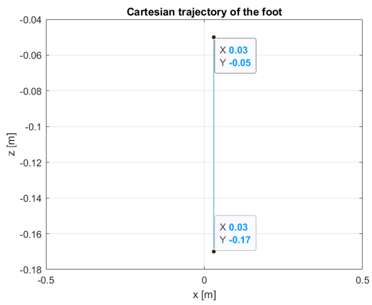
(f) Angles in leg joints



(g) Torque in leg joints



(h) Torque-speed curve of the leg and each joint



(i) Trajectory of the foot in Cartesian coordinates

Figure 1: Results from optimization for gear ratio 6:1, mas of the leg 1.0 kg, link length 0.11 m, final velocity 45 $\frac{rad}{s}$

For the parameters set above, the trajectory optimization was performed. The results are represented on Fig. 1. Figures 1a, 1b 1c show the ground reaction force, velocity, and position of the leg respectively. On the figures from 1d to 1g there are present the acceleration, velocities, position and torques of the joints (q1 is the hip, and q2 is the knee). Fig. 1h represent the torque-speed curve of the gearbox with the used motor and the relation between torque and speed of each joints. The trajectory of the leg's foot is represented on the Fig. 1i, which was calculated using inverse kinematics and the obtained angles of joint in each grid point. From the obtained results the height of the jump is equal 0.665 m. Based on the results, it can be concluded that the optimization works properly and for the selected parameters it is possible to generate trajectory in the launch phase of the jump.

2.3.2 Reduction 8:1

In order to check the performance of the leg, another simulation was executed using a different set of parameters. In this optimization, the following parameters were chosen:

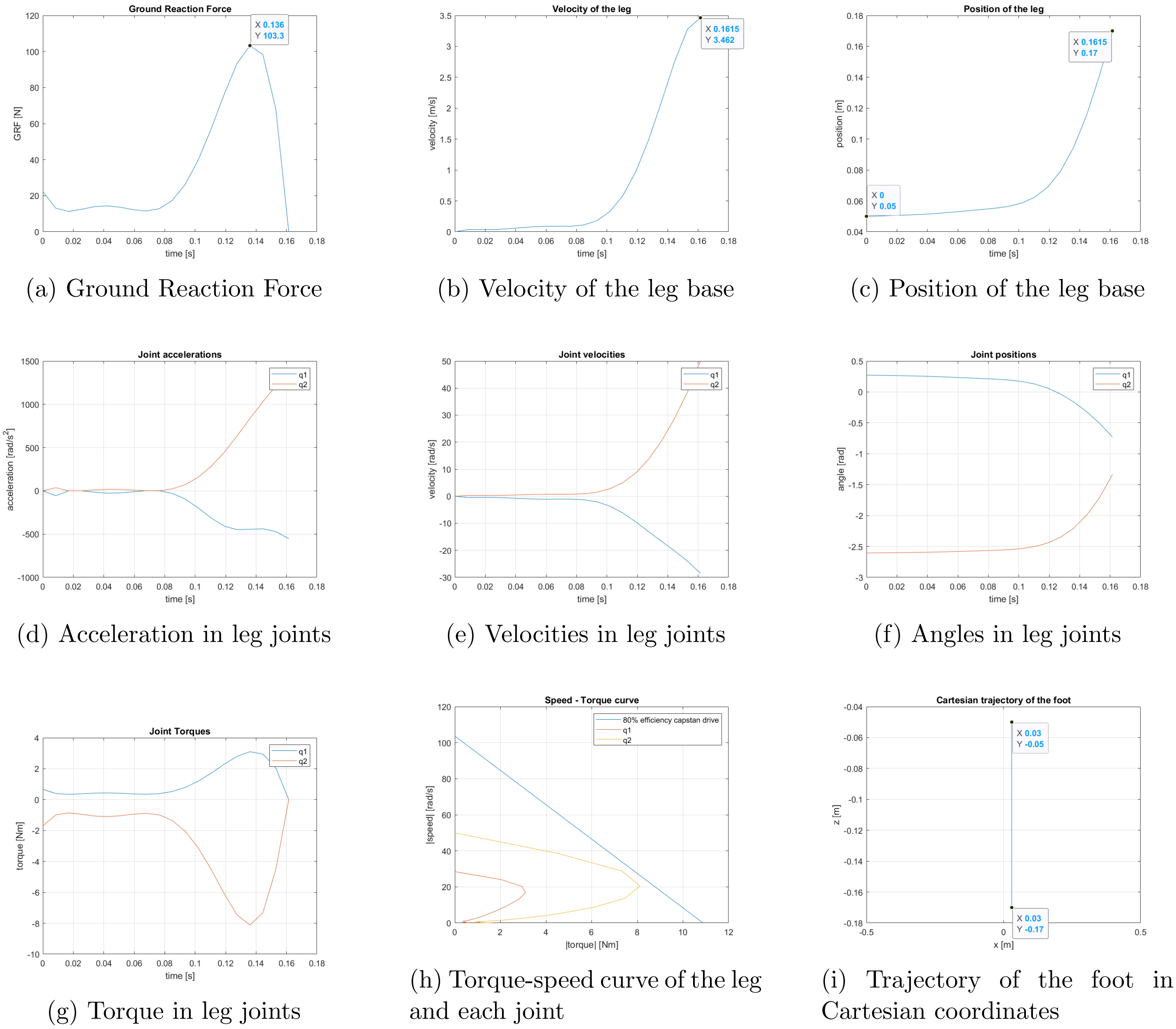
- gear ratio: 8:1
- links length: 0.11 m
- mass of the leg: 1,2 kg

Based on the parameters and the knowledge about parameters of the chosen motor, and assumption that the capstan drive has efficiently of 80% the following parameters can be calculated:

- maximum output speed of the gearbox: $103.67\frac{rad}{s}$
- maximum output torque of the gearbox: 10.88 Nm

As previously, the initial and final position and velocity of the leg's foot was set as following:

- initial value in step 0:
  - position: x = 0.03 m, z = -0.05 m
  - velocity: $0\frac{rad}{s}$
- final value in step N:
  - position: x = 0.03 m, z = -0.17 m
  - velocity: $50\frac{rad}{s}$



(a) Ground Reaction Force

(b) Velocity of the leg base

(c) Position of the leg base

(d) Acceleration in leg joints

(e) Velocities in leg joints

(f) Angles in leg joints

(g) Torque in leg joints

(h) Torque-speed curve of the leg and each joint

(i) Trajectory of the foot in Cartesian coordinates

Figure 2: Results from optimization for gear ratio 8:1, mas of the leg 1.2 kg, link length 0.11 m, final velocity 50 $\frac{rad}{s}$

For the parameters set above, the trajectory optimization was performed. The results are represented on Fig. 2. From the obtained results the height of the jump is equal 0.781 m. Based on the results, it can be concluded that the optimization works properly and for the selected parameters it is possible to generate a trajectory in the launch phase of the jump. The obtained height of the jump for the gear ratio 8:1 is 0.116 m greater than for the gear ratio equal 6:1. It can be concluded that for the chosen motor, to perform higher jump, more output torque is needed.

3 Capstan Drive

The purpose of this task is to design, manufacture, and test a capstan drive. The capstan drive will consist of a brushless motor, capstan, cable, and tensioning system. The mechanism should provide zero backlash, low inertia, and backdrivability,which are all required in a quadruped robot. All parts of the capstan drive will be 3D printed, which will reduce the cost of production and comparison of multiple prototypes

3.1 Design

At the beginning, a 3D model of the capstan drive was designed in Fusion 360, which provides all the necessary tools and rendering mode for better visualization. In the basic form the program is free, but in this project an educational license was used which provides all functionalities.

The difficulty of the design is that most types of actuators can be designed using toolboxes which generate most of the elements automatically for a given set of parameters. This type of actuator is much less popular than the others, which translates into the lack of ready-made solutions and guides on how to make this type of actuator.

3.1.1 Capstan drive with reduction 6:1

The main goal of the first design was to design the capstan drive as small as possible without compromising the quality of any component. The important fact is that it should be designed in such a way that it can be 3D printed and assembly using hexagonal screws with standard dimensions.

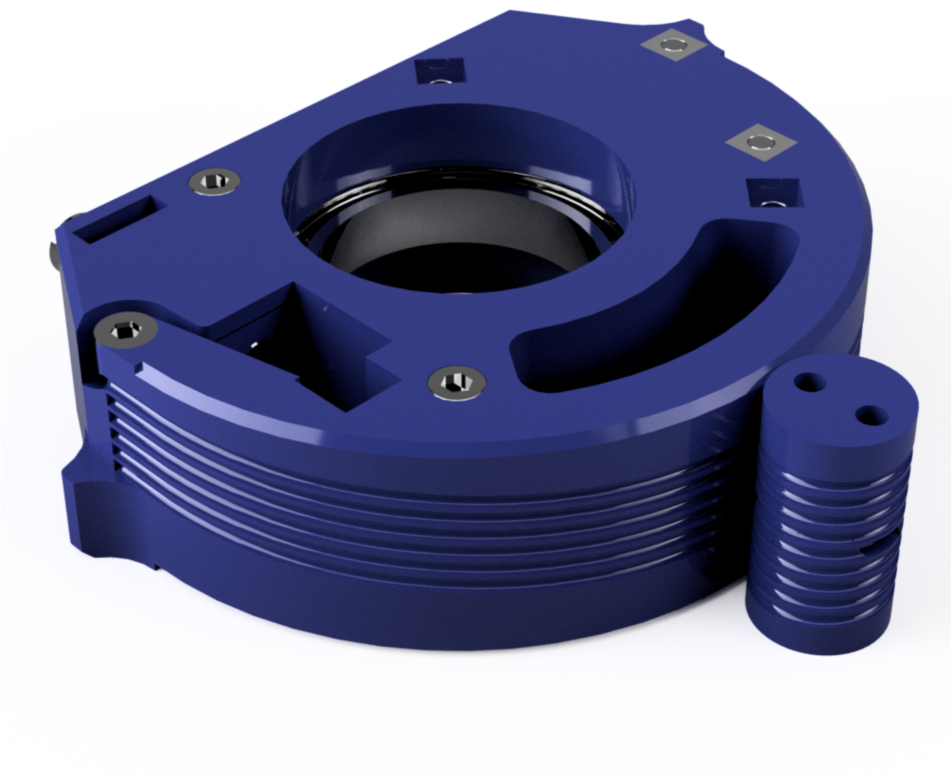
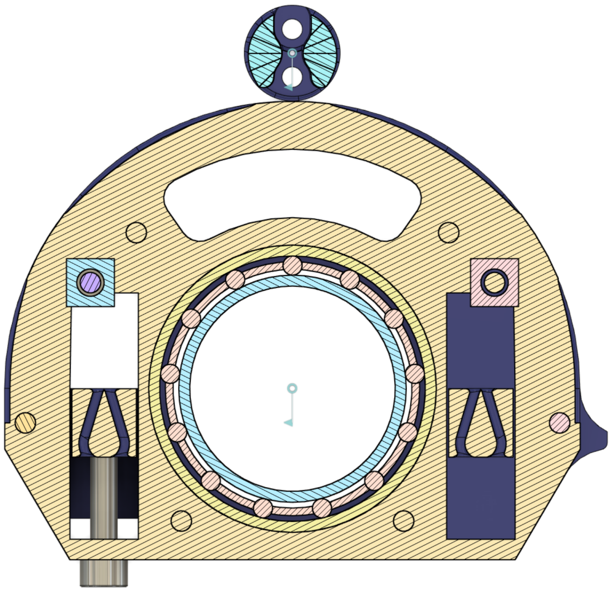


Figure 3: 3D model of a capstan drive with gear ratio 6:1

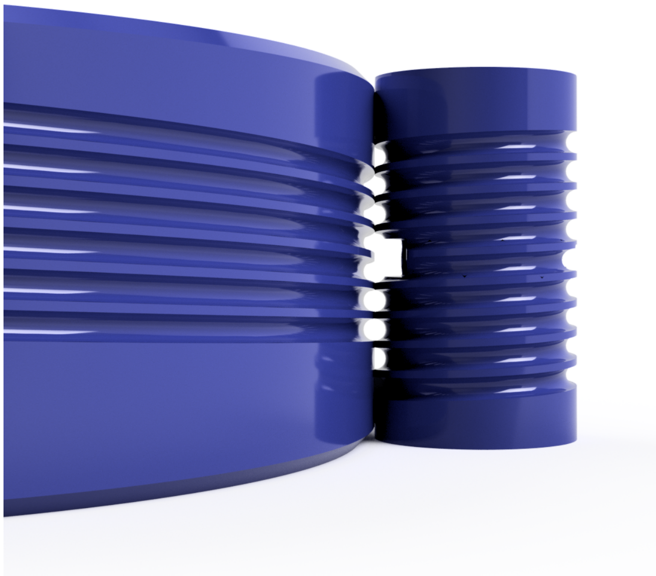
A capstan drive with the following parameters was designed:

- gear ratio: 6:1
- input capstan diameter: 14 mm
- output capstan diameter: 84 mm
- height: 25 mm

The 3D model of the design is presented on Fig. 3. The capstan drive consists of 4 printed parts, 10 hexagonal screws, 10 square nuts, one MGK 6806 bearing, and DM20 rope (with diameter equal 1mm). It has an angle range of 180 degrees.



(a) Cross-selection of the 3D model



(b) Point of contact of the input and output capstan (with omitting DM20 rope)

Figure 4: Cross-selection of the 3D model on the left and a point of contact of the input and output capstan

One of the most important parts in this type of actuator is to design and manufacture a lightweight, small, durable, and easy to assemble tensioning system. Fig. 4a shows the cross-selection of the 3D model. It shows two tensioning systems on each side, where the rope is tightened by tightening the screw. In the middle of the cross-selection there is a bearing, which enables easy assembly.

Another important part of the capstan drive is the point of contact of the input and output capstans, which is presented on Fig. 4b. Because there are paths for the winding rope on both input and output capstans. It is important that at every moment of rotation these paths should match ensuring smooth winding and unwinding of the rope.

3.1.2 Capstan drive with reduction 8:1

To ensure more torque, there was also designed a capstan drive with higher gear ratio. The elements of the design are similar as in the capstan drive with gear ratio 6:1. Because the input capstan has only 14 mm, the only way to get a greater gear ratio was to increase the diameter of the output capstan.

The new capstan drive has the following parameters:

- gear ratio: 8:1
- input capstan diameter: 14 mm
- output capstan diameter: 112 mm
- height: 30 mm

The new capstan drive consists of 4 printed parts, 10 hexagonal screws, 10 square nuts, one MGK 6810 bearing, and DM20 rope (with diameter equal 1mm). It has an angle range of 180 degrees. Comparison of the capstan drive with gear ratio 6:1 and 8:1 shows Fig 5.

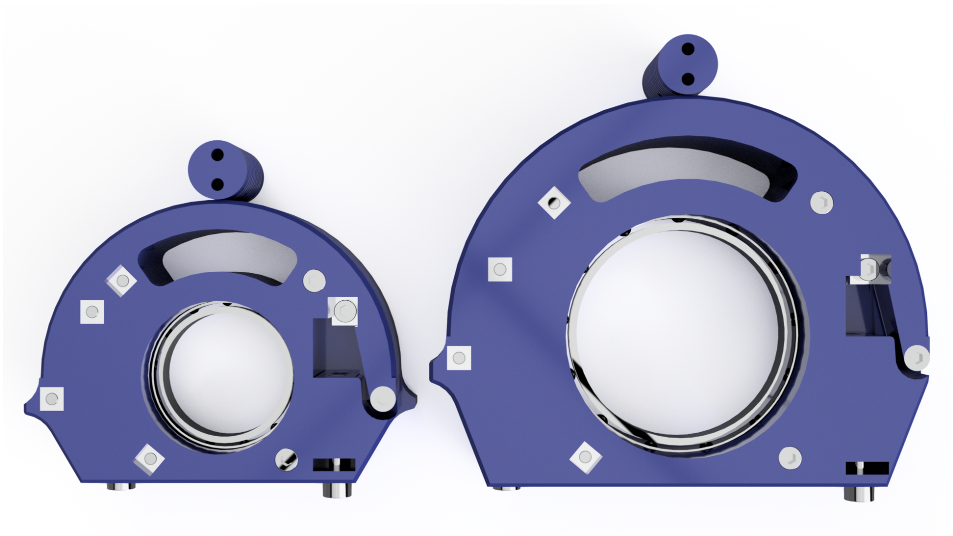


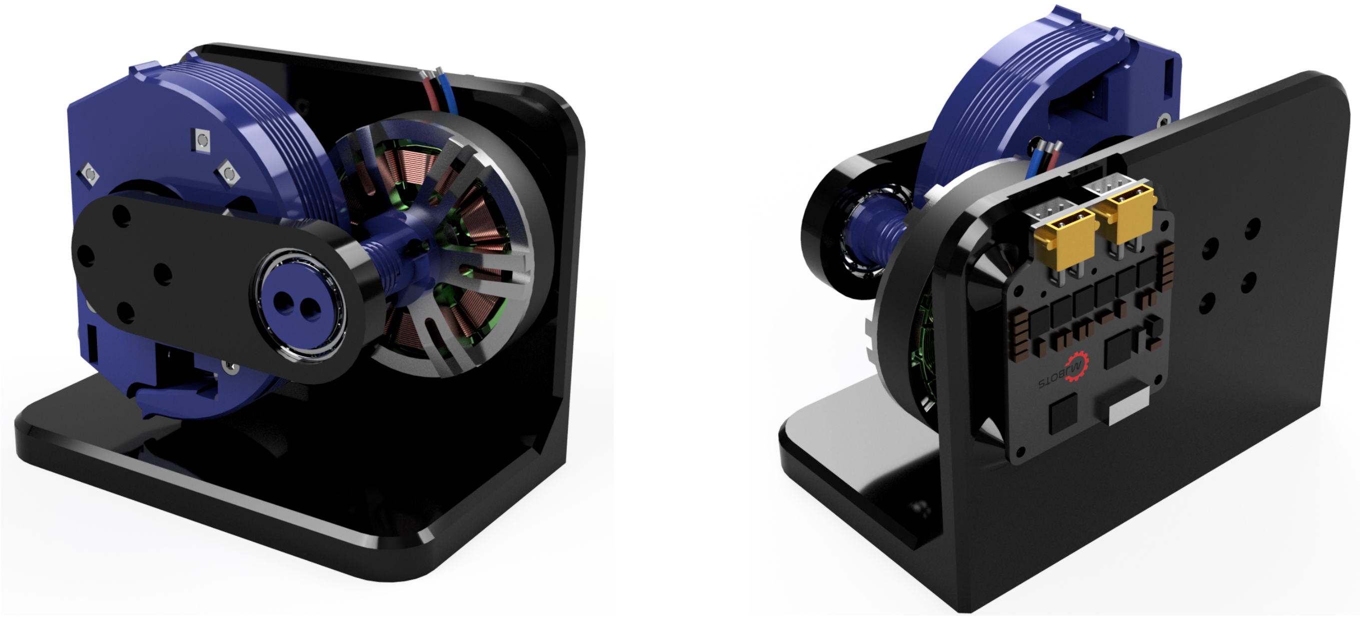
Figure 5: Comparison between capstan drive with gear ratio 6:10 and gear ratio 8:1

3.1.3 Test Stand

In order to assembly the capstan drive and perform a test, a simple test stand was designed. It consist of the following parts:

- capstan drive with gear ratio 6:1
- mj5208 brushless motor
- moteus 4.3 brushless motor controller [4]
- base consists of 3 printed elements
- one 6803 bearing which holds the input capstan

The designed 3D model of the test stand is presented on Fig. 6.



(a) front of the test stand (b) back of the test stand

Figure 6: 3D model of designed test stand

3.2 Manufacture

The designed test stand was manufactured and assembled. All necessary parts were 3D printed using PETG filament. To prepare the element to print, a PrusaSlicer software was used, which is a free software provided by Prusa company. A Prusa mk3s+ 3D printed was used to print all elements. The main difficulties in the assembly were to correctly wind and tension the rope, which is necessary for proper operation of the capstan drive. The assembled test stand shows Fig. 7.

(a) front of the test stand (b) back of the test stand

Figure 7: Assembled test stand of capstan drive with gear ratio 6:1

3.3 Tests

In order to perform tests a program in Python was implemented. The main assumptions of the test were to check the resistance of the capstan drive to dynamic changes in the direction of rotation, which resulted in impulse changes in the torque. A control loop were implemented in which the following subsequent steps were performed:

1. move to position "start" with constant speed 120 RPM
2. move to position "final" with constant speed -120 RPM
3. repeat the procedure

where the "start" position means 0 degrees on the output and "final" position means 180 degrees on the output. Therefore, there were changes in speed by 240 RPM in a fraction of a second. It is important to notice that the described values are given in relation to the output capstan. The velocities on the input capstan (in the motor) are 6 times greater than the given speeds. After the tests, the rope was still taut and none of the components were damaged. The mechanism provides zero backlash, low inertia, and backdrivability, which are all required in a quadruped robot. The tensioning system is small, durable, and works as expected.

4 Plans for further development

Based on the obtained results, there is a plan to design and manufacture the entire 2 DoF leg using one of the designed capstan drives. Next, using the results from the optimization, the control algorithm will be implemented. The purpose of the algorithm will be to control the leg in order to perform a jump as close as possible to the theoretical obtained from the simulation [2], [7].

5 Summary

As a result of the project, a two capstan drive with different gear ratios was designed. One of them was manufactured and tested. Additionally, a nontrivial optimization task was performed to generate a trajectory of the leg movement during a vertical jump. The optimization also allows to select all necessary parameters of the leg as well as providing information of the forces and speed needed to perform a vertical jump.

References

[1] Anirban Mazumdar, Steven J. Spencer, Clinton Hobart, Jeffrey Dabling, Timothy Blada, Kevin Dullea, Michael Kuehl and Stephen P. Buerger. Synthetic Fiber Capstan Drives For Highly Efficient, Torque Controlled, Robotic Applications.

[2] Dingkui Tian, Junyao Gao, Xuanyang Shi, Yizhou Lu and Chuzhao Liu. Vertical Jumping for Legged Robot Based on Quadratic Programming.

[3] E.H. Doha, A.H. Bhrawy, M.A. Saker. Integrals of Bernstein polynomials: An application for the solution of high even-order differential equations.

[4] J. Pieper. mjbots github. `https://github.com/mjbots`. Access date: October 2021.

[5] Jemin Hwangbo, Vassilios Tsounis, Hendrik Kolvenbach and Marco Hutter. Cable-Driven Actuation for Highly Dynamic Robotic Systems.

[6] Nicholas P. Linthorne. Analysis of standing vertical jumps using a force platform.

[7] Yanran Ding, Hae-Won Park. Design and Experimental Implementation of a Quasi-Direct-Drive Leg for Optimized Jumping.