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Dynamic Modelling of Trajectory Tracking Pioneer 3-DX Differential Drive Mobile Robot

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Abstract

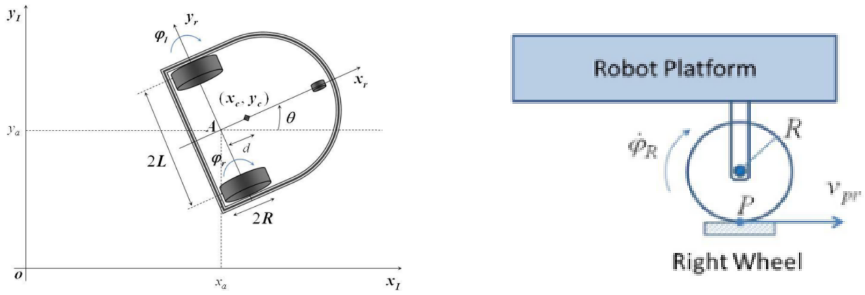
The goal of the project is Dynamic Modelling of Differential Drive Wheeled Mobile Robot (WMR) using Lagrangian formalism and design sliding mode(SM) controller for smooth trajectory tracking, the movement of WMR is described by two non-holonomic constraint equations, obtained by two main assumptions these are no lateral slip motion and pure rolling constraint. the position and orientation of the robot model are controlled by designed inner and outer SM controllers. The WMR is modeled using MATLAB Simulink environment and implemented in Pioneer 3-DX mobile robot.

This project was successful. All the major goals were fulfilled.

1 Introduction

Differential-drive Wheeled Mobile Robots (DDWMR) are types of robots that can travel from one place to the other without support from a human being. It has a special feature of moving easily within predefined work volume to reach final desired goals, not in common with most industrial robots. This ability of moving is suitable for a huge application in unstructured and structured environments. It has two independent derivable wheels attached to a common axle. These two wheels were individually driven and the one castor wheels were important for stability and balance. A differential-drive WMR is presented in Figure 1(a). The movement WMR for the differential drive is described by two non-holonomic constraint equations, which are obtained by two main assumptions:

1. No lateral slip motion: The WMR can move only forward and backward but not side-ward.
2. Pure rolling constraint: Individual wheel retains a single interaction point P with the ground as shown in Figure 1(b).



(a) Wheeled mobile robot for differential drive with it's coordinate system

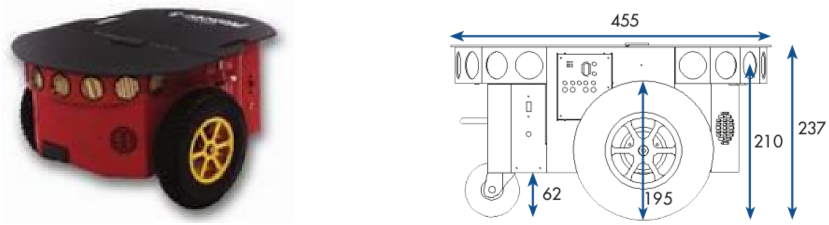
(b) Pure rolling constraint of wheeled mobile robot

Figure 1: Wheeled mobile robot [2]

There is an actual difference between the real robot and its measured robot model, so this initiates the designing of the control system. These differences occur due to different external and internal factors. The presence of the above factors makes designing the appropriate control law is a very difficult task, to solve those mentioned problems sliding mode control is an appropriate robust control method. Sliding mode control(SMC) is a type of nonlinear control, which has incredible behavior of accurateness, easy tuning, and robustness. SM control system designed which forces the system states at certain surface named a sliding surface. Once it reaches the sliding surface SM control preserves the system states close to on the sliding surface. It has two parts, the first part is the design of the sliding surface and the second part is the choice of the appropriate control law. SMC has many advantages among those is, the system behavior is governed by switching surface and the second category is the response of the system not sensitive to internal and external factors i.e uncertainty, internal and external disturbances. The main focus of this project is to formulate the "trajectory tracking control of differential drive mobile robots" by developing a mathematical model and designing a relevant controller based on SM controler. A differential-drive WMR has two controllers, kinematics controller, and dynamic controllers. both The kinematics controller and dynamic controller were designed using SM, the two controllers were designed separately and combined together. Finally the model will simulate in MATLAB Simulink and implemented in Pioneer 3DX mobile robot.

2 Pioneer 3-DX mobile robot

Pioneer 3-DX is a small lightweight two-wheel two-motor differential drive robot ideal for indoor laboratory or classroom use. The robot comes complete with front SONAR, one battery, wheel encoders, a microcontroller with ROS firmware, and the Pioneer SDK advanced mobile robotics software development package. Pioneer research robots are the world's most popular intelligent mobile robots for education and research. Their versatility, reliability and durability have made them the preferred platform for advanced intelligent robotics. Pioneers are pre-assembled, customizable, upgradeable, and rugged enough to last through years of laboratory and classroom use.



(a) pionner 3-DX mobile robot system

(b) pionner 3-DX mobile robot Dimensions(mm)

Figure 2: Pionner 3-DX mobile robot [1]

both Mathematical modeling and controller design is based on the Pioneer 3-dx parameters which include its dimensions, the mass of the body, mass of the wheels, the distance between the wheel, and motor actuator modeling.

3 Mathematical Modeling

3.1 Kinematic Model of Differential Drive Mobile Robot.

Kinematics deals with the movement of mobile robot systems that do not take into account the forces which are affecting the motion. It allows computing the position and orientation of the robot relative to the global frame. Representations of the kinematics model of a wheeled mobile robot using linear and angular velocities are shown below.

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \tag{1}$$

3.2 Dynamic model

Dynamics deals with the motion of a system taking into consideration the several forces which affect its motion.The dynamic model of the robot is vital for simulation study of the robot and the development of several motion and control systems. The Lagrange approach is used to derive the equations of motion of a given system, with consideration of kinetic energy since the mobile robot moves only in the horizontal plane the potential energy becomes zero, the only thing is finding the kinetic energy. The WMR dynamics are expressed as a function of the right and left wheel angular velocities, the robot angular velocity, and the driving motor torques. The equations of motion of dynamic model with actuator dynamics used for simulation is shown below

$$\left(m_t + \frac{2I_w}{R^2}\right)\dot{v} - m_c d\omega^2 v = \frac{NK_t}{RR_a}(V_1 + V_2 - 2K_b\omega) \tag{2}$$

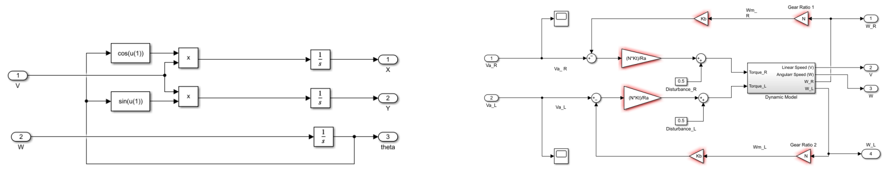
$$\left(I + \frac{2L^2}{R^2 I_w}\right)\dot{\omega} + m_c d\omega v = \frac{NLK_t}{RR_a}(V_1 - V_2) \tag{3}$$

Kinematic and dynamical representation of DDWMR in MATLAB/Simulink is shown in Figure 3(a) and (b)

4 Controller Design

To have worthy motion control performance the controller divided into two phases as it shown in Figure 4:

1. an inner loop , It is also called dynamic-level control of a mobile robot;
2. an outer loop to control the posture of the robot. It is also called kinematic-level control of a mobile robot. .



(a) representation of the kinematic model of DDWMR system

(b) representation of the dynamic model of DDWMR

Figure 3: Kinematic and dynamic modelling

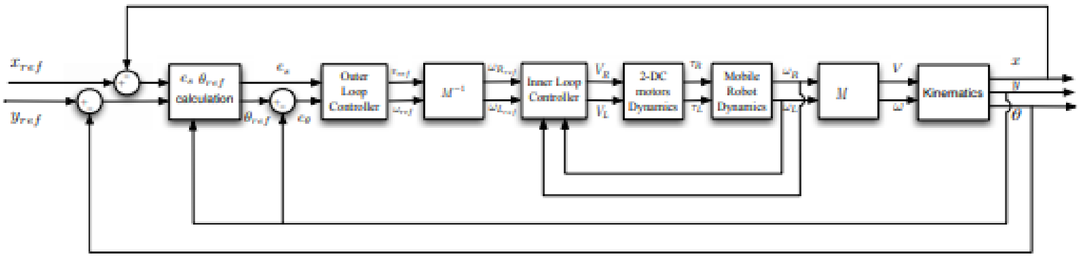


Figure 4: dynamic-level control of a mobile robot

4.1 Slide mode controller

Sliding mode control has a switching surface called the sliding surface because a control trajectory has again if the state trajectory of the system is "above" that surface and a different gain if the trajectory drops "below" that surface. Secondly, it retains the plant's state trajectory on this surface for all consequent times

To develop a control law based on the sliding mode contains two parts is designing an appropriate switching surface to make the system on that switching surfaces and next of designing of a suitable control law

- **Switching surface :** To developed an appropriate control law choosing a suitable switching surface is unquestionable, the switching surface s(x) = 0 is determined by finding the intersection of k sliding surfaces depending on the system the switching surface is either linear or non linear.
- **Existence condition :** After designing of the switching function the next procedure is the guarantee of the existence condition of a sliding mode controller.
- **Reaching condition :** It is a condition that for a system to reach the sliding surface with finite time.
- **Reaching time :** The time needed to reach on the sliding surface is called reaching time.

5 Summary

The project idea has been developed in three milestones:

1. First Kinematic Model of Differential Drive mobile Robot is designed and using the kinematic model open-loop behavior of designed model is checked using MATLAB/Simulation block.
2. In the next month, the dynamical model of the robot and slide mode controller is built, checked the built model using different paths to learn the tracking error of the position and orientation of the model and using the result get from tracking re-tune the parameters of the slide mode controller to get best tracking.
3. Finally, after the model and controller are effective in Matlab/Simulink the model is implemented on the PIONEER 3Dx robot and based on the result get from the field the controller is re-tune to get the best tracking.

6 Implementation

First, the mathematical model of the robot is calculated on paper, taking into account its mass, external force and actuator, then it is built in MATLAB simulink, the parameters of the controller are re-tuned until the best result is obtained from the model. The performance is checked for different trajectories using different input trajectories and disturbance, and it is effective to track the x-axis, y-axis and orientation between axis. After the model and controller worked in MATLAB, the next step was to implement in Pioneer 3DX mobile robot, when it is implemented in the real field, the model needs to be re-tuned and some modeling parameters fixed, because the assumptions initially taken in mathematical modeling do not work in the real world, therefore, the parameters of the controller have to be re-tuned to obtain the best best trajectory tracking.

MATLAB representaion of the plant shown in Figure 5 kinamatic model , dynamic model , controller are built in inside the blocks.

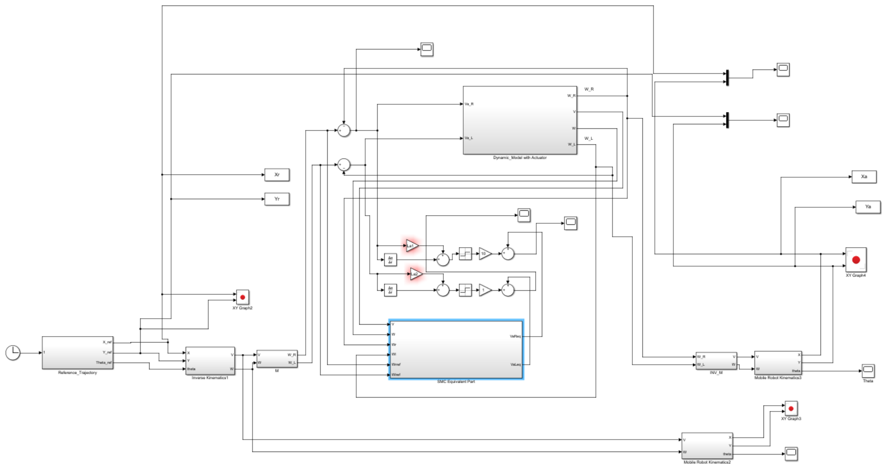
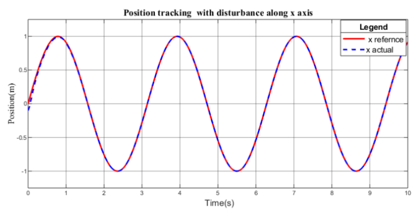


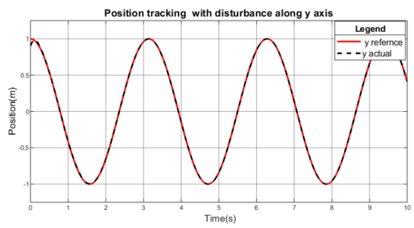
Figure 5: Simulink diagram

7 Results

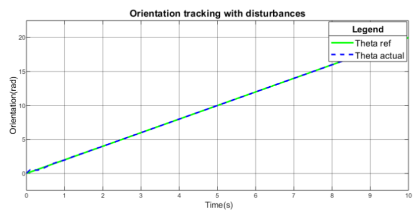
Simulation result of the SM control based on dynamical model DDWMR Position and orientation tracking of different trajectories.



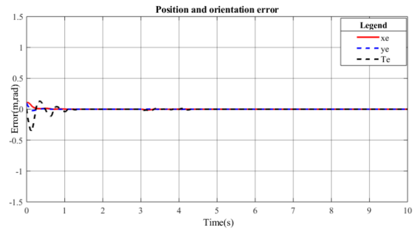
(a) X axis position tracking



(b) Y axis position tracking



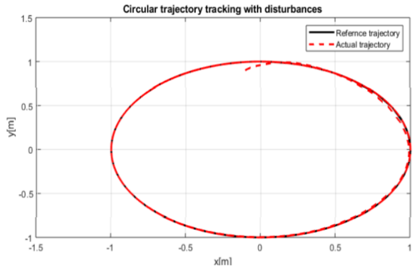
(c) orientation tracking



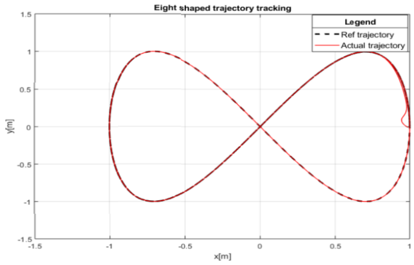
(d) position and orientation error

Figure 6: Model position and orientation Tracking along different axis and its error

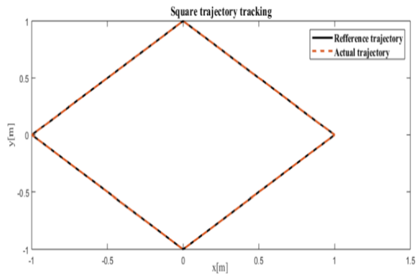
from Figure 6 (a) (b) and (c) it can be seen that the actual trajectory tracks the reference trajectory position and orientation from the initial up to the final within the given simulation time 10 second and also in Figure 6 (d), it can be observe that there is tracking error initially in both position and orientation but converges to zero immediately, the tracking error of orientation is greater than tracking error of position it is because of that the robot may initially put in different direction due to this the orientation error is greater than the position error but the controller reduce it to zero within a second.



(a) circular shape trajectory tracking along x-y axis



(b) eight shape trajectory tracking along x-y axis



(c) Square shape trajectory tracking along x-y axis

Figure 7: Trajectory tracking in X-axis and Y-axis

from Figure 7 it can be shown that the the given sliding mode controller perfectly track different trajectories i.e circular, eight, and square shaped trajectories. it shows the given controller is an effective, when the path is changed and having different disturbances.

Practical Result : circular trajectory using Pioneer 3-DX robot can be seen using this link : http://www.youtube.com/shorts/Od-612372xY?feature=share

8 Conclusion

This project has been successful. All the goals were accomplished. The control law tracks the given different trajectories position and orientation in the x-axis and y-axis effectively and makes the error between actual and track trajectory converge to zero in a short time this indicates that the given controller performance is accurate for the given system.

References

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[3] J. Xue, S. Dong, and J. Yan, “Sliding mode variable structure based path follow- ing control of agricultural robots with differential drive,” IFAC-PapersOnLine, vol. 51, no. 17, pp. 455–459, 2018.
[4] E. S. Elyoussef, E. R. De Pieri, U. F. Moreno, and M. Jungers, “Super-twisting sliding modes tracking control of a nonholonomic wheeled mobile robot,” IFAC Proceedings Volumes, vol. 45, no. 22, pp. 429–434, 2012.