

Akustyczne czujniki odległości

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Wybrane zagadnienia robotyki

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Niniejszy dokument zawiera materiały do wykładu dotyczącego wybranych zagadnień robotyki. Jest on udostępniony pod warunkiem wykorzystania wyłącznie do własnych prywatnych potrzeb i może on być kopiowany wyłącznie w całości, razem z niniejszą stroną tytułową.

Spis treści

- 1 Ultrasonic sensing
 - Electrostatic transducers
 - Piezoelectric transducers
 - Features of ultrasonic sensing
- 2 Reflection angle for a set of mirrors
 - Interpretation of Equations
 - How it is exploited?
 - What a robot can see by using ultrasonic range finders
- 3 Echo threshold detection method
 - Principles of operations
 - Measurements—main features

Principle of distance measurement



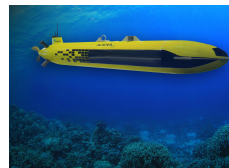
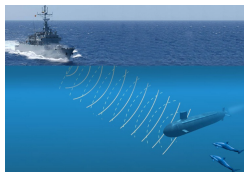
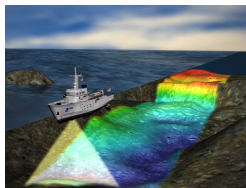
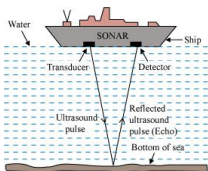
Measurement principle:

$$s = \frac{v_a \Delta t}{2}$$

Δt – interval of time measured between moments of an signal emission and echo detection.

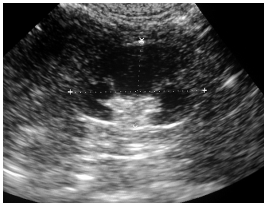
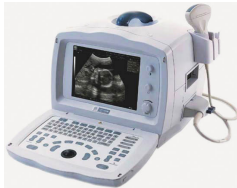
v_a – speed of sound in air in standard conditions for temperature and pressure i.e. temperature: 20 °C, pressure: 1 atm (101 325 Pa) (according to the National Institute of Standards and Technology) — 343 $\frac{m}{s}$.

Time of Flight → Distance Measurement



The technique of time measurement is widely used in many applications such as objects detection and tracing or surface scanning and imaging.

Distance Imaging



Very important application is medical ultrasonography used for diagnostic. This technique combined with information of signal strength makes it possible to 2-D or 3-D imaging.

Development of Tesla car



Some time ago it was announced that in a new Tesla car the main source of information for autopilot will be a radar, not the vision system. The radar will be able the autopilot to deliver a 3D depth map.

Tesla radar and autopilot helped to avoid a car accident



A driver of the red car noticed that a car ahead of his own started braking but he did not started braking his car.

Tesla radar and autopilot helped to avoid a car accident



The driver of the red car wanted to pass by the car head of his own and, then, started the maneuver. At this moment, in the car where this video was shot, Tesla autopilot turned on warning.

Tesla radar and autopilot helped to avoid a car accident



The black SUV which was ahead of the red car was braking hard. It was noticed too late by the driver of the red car. Therefore, he could not avoid collision. But the car equipped with Tesla autopilot slowed and stopped without any problem.

Range finders for robots



In robotics, ultrasonic range finders are widely used as distance sensors.

Speed of acoustic wave

Dependency on temperature:

$$v_a = 331.5 \sqrt{1 + \frac{T}{273.15}} \left[\frac{m}{s} \right]$$

T – the temperature is expressed in $^{\circ}C$.

Approximated dependency:

$$v_a = 331.5 + 0.606 T$$

Types of sounds

...	–	20Hz	Infrasound
20Hz	–	20kHz	Hearing sound
20kHz	–	10GHz	Ultrasound
10GHz	–	...	Hipersound

Who can hear it?

$< 4kHz$ — herring



$< 22kHz$ — dog



$< 160kHz$ — dolphin and whale



$< 180kHz$ — alewife



$< 200kHz$ — bat



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Ultrasonic sensing

Reflection angle for a set of mirrors
Echo threshold detection method

Electrostatic transducers

Piezoelectric transducers
Features of ultrasonic sensing

How it started



Polaroid SX-70

Author: Timmy Toucan

Source: http://camerapedia.wikia.com/wiki/Polaroid_SX-70

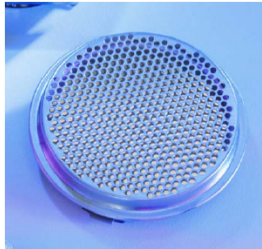
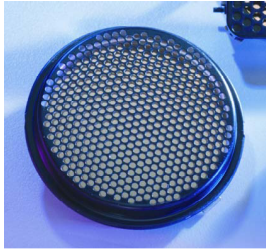
Licence: <http://creativecommons.org/licenses/by-nc-sa/2.0>

Pioneer 3-AT



Electrostatic ultrasonic transducers are one of the main part of the basic equipment of Pioneer and PeopleBot robots.

Transducers series 600

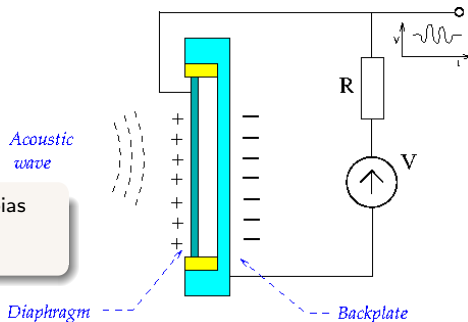


Source: <http://www.senscomp.com>

Operation of ultrasonic electrostatic transducers is based on the same principle as operation of electrostatic loudspeakers and microphones.



Electrostatic microphone (capacitor microphone)

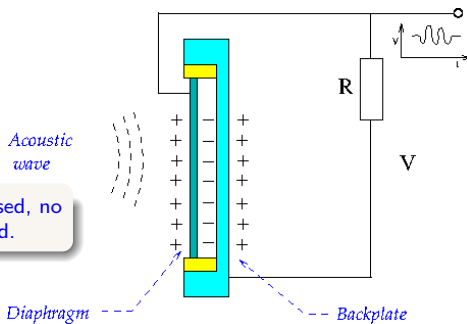


$$V_{out} = \frac{Q}{C}$$

Voltage is required to bias the diaphragm of the condenser microphone.

The charge of the capacitor is practically constant and the voltage across it changes to reflect the change in capacitance caused by movement of the diaphragm. The voltage across the capacitor varies above and below the bias voltage.

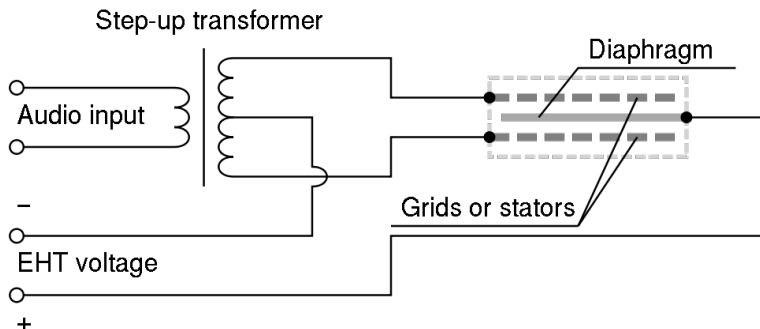
Electrostatic microphone (capacitor microphone)



When an electret is used, no bias voltage is required.

The charge of the capacitor is practically constant and the voltage across it changes to reflect the change in capacitance caused by movement of the diaphragm. The voltage across the capacitor varies above and below the bias voltage.

Electrostatic loudspeakers

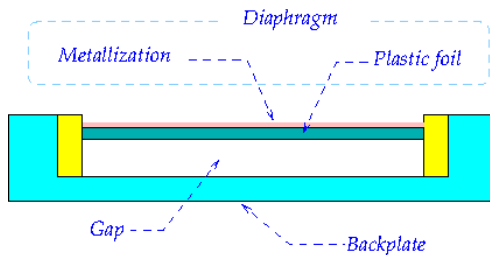


Author: Akilaa

Source: http://commons.wikimedia.org/wiki/File:Electrostatic_loudspeaker.svg

License: © (public domain)

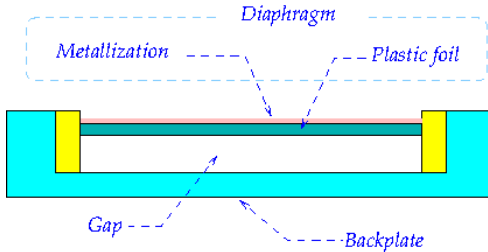
Diaphragm



Thickness of the plastic foil is usually in the range of 3 – 40 μm . It is made of styroflex, mylar or similar materials.

The foil is metalized on one side by a very thin layer of gold, copper or aluminium

Diaphragm

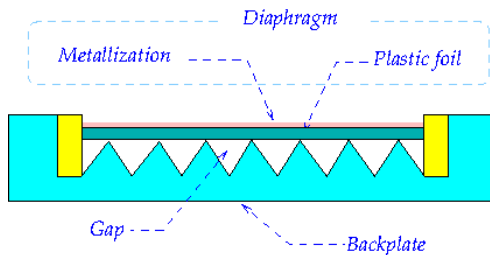


One of the most important problem consists in keeping the working gap as small as possible over the whole transducer diaphragm. It is very difficult to achieve by tensioning the diaphragm.

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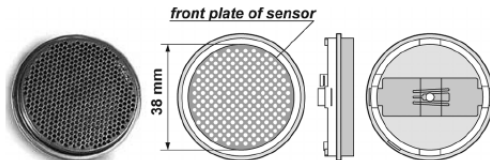
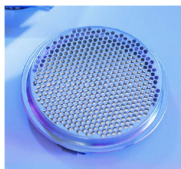
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Diaphragm



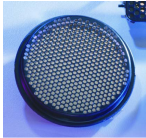
The problem can be solved by dividing the sensitive area into a large number of smaller elementary transducers. It can be accomplished by a system regularly (or irregularly) arranged grooves.

Transducers produced by SensComp



Diaphragms of transducers distributed by SensComp are made with elastic foil finished with gold. A transducer diaphragm is placed into a stainless steel housing. The assembly comes complete with a perforated protective cover.

Transducers produced by SensComp



There are also transducers with an open housing. To enable the transducer to operate in a harsh environment an Propylene coating is added to a diaphragm.

The open face construction of SensComp's Series 600 Transducer minimizes the potential of dust and powdered material collecting on the front face of the transducer. The added protection of the Parylene conformal coating makes this transducer splash resistant and able to operate more efficiently in harsh chemical environments containing organic and inorganic solvents. Additionally, the Parylene coating provides extended protection against corrosion and mechanical abrasion.

Electrostatic transducers

The range finders used by Polaroid were based on electrostatic transducers.

Their advantages:

- strong signal,
- narrow beam,
- long range,
- their work as transceivers.

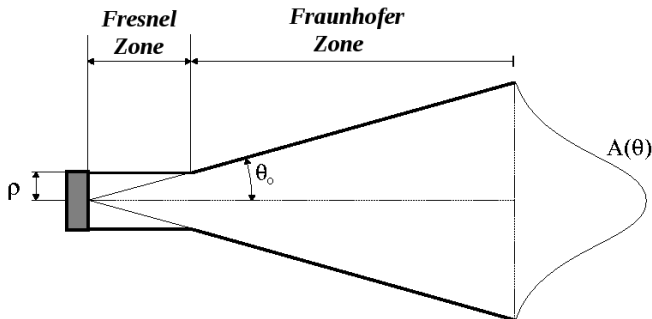
Their disadvantages:

- large size,
- high voltage is required to generate excite pulses and to receive bias voltage is needed.
- they are noisy.

Currently, this type of transducers is distributed by firm SensComp.



Zones of wave propagation



Rysunek: An area of ultrasonic signal propagation

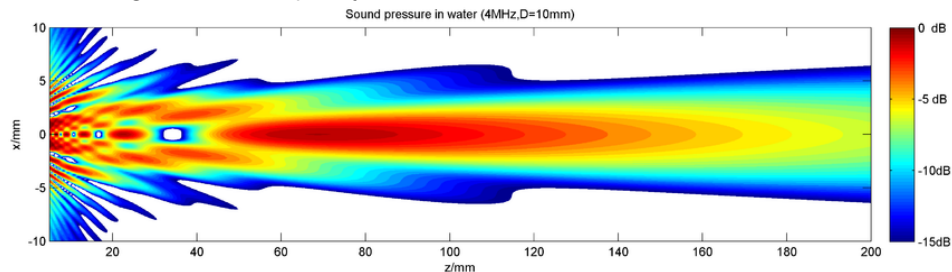
$$\theta_0 = \arcsin \frac{0.61\lambda}{r}$$

– the length of Fresnela zone (~ 6 cm)

– beam width ($\sim 11^\circ$)

Fresnel and Fraunhofer zone

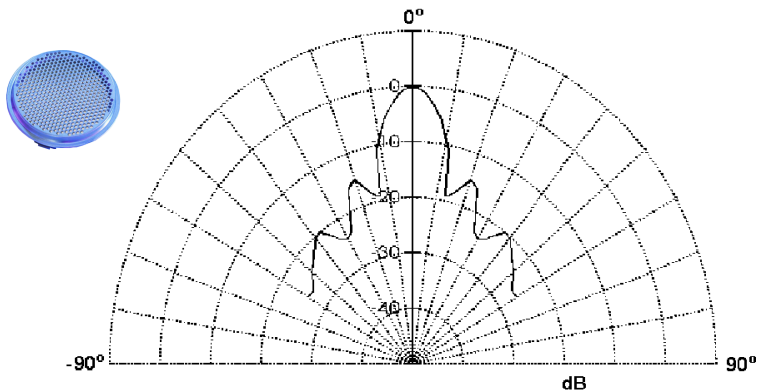
Acoustic pressure near an ultrasonic sender of diameter 10mm which emits a signal of the frequency 4MHz.



Author: Michael Lenz

Source: http://en.wikipedia.org/wiki/File:Soundfield_Water_4MHz_TransducerRadius5mm.png#filelinks

Directivity pattern



Rysunek: An example of directivity pattern of Polaroid transducer at 50 kHz

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- Features of ultrasonic sensing

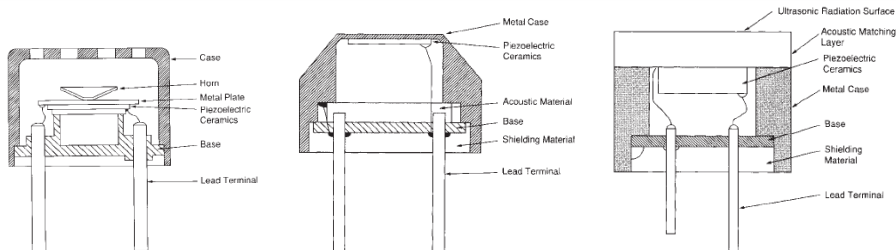
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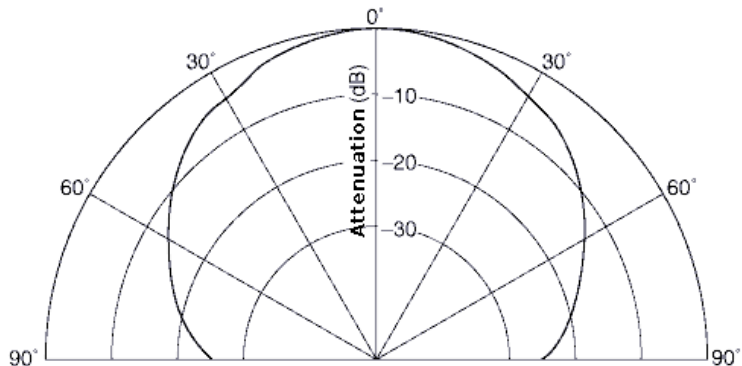
Construction



Examples of piezoelectric materials:

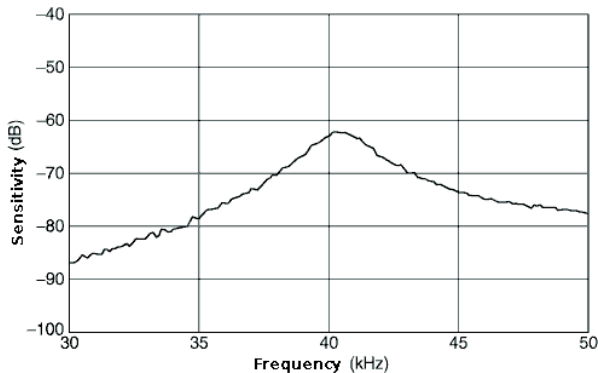
- Quartz SiO_2
- Zirconate piezoceramics PTZ-5A
- Lead-niobate piezoceramics PbNb_2O_6
- Piezopolymer PVDF
- Piezopolymer P(VDF-TrFE)

Ultrasonic transducers



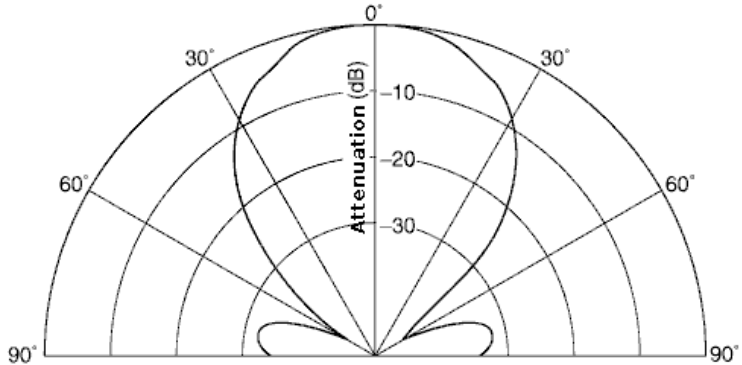
Rysunek: Directivity pattern of a piezoelectric transducer MA40S4R produced by Murata

Ultrasonic transducers



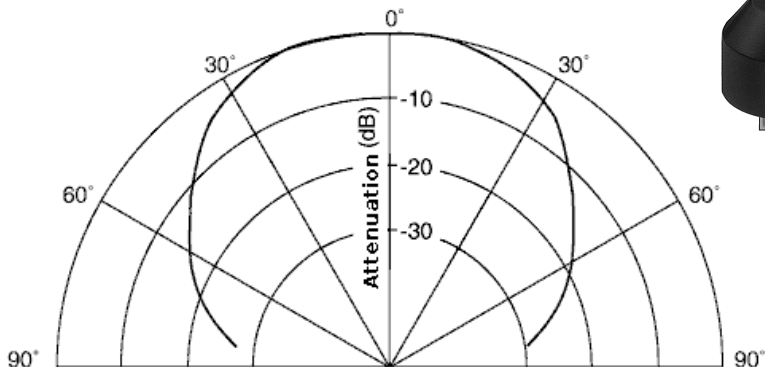
Rysunek: Frequency characteristic of a piezoelectric transducer MA40S4R produced by Murata

Ultrasonic receiver



Rysunek: Directivity pattern of a piezoelectric receiver MA40B8R produced by Murata

Ultrasonic transducers (transmitter/receiver)



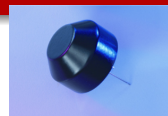
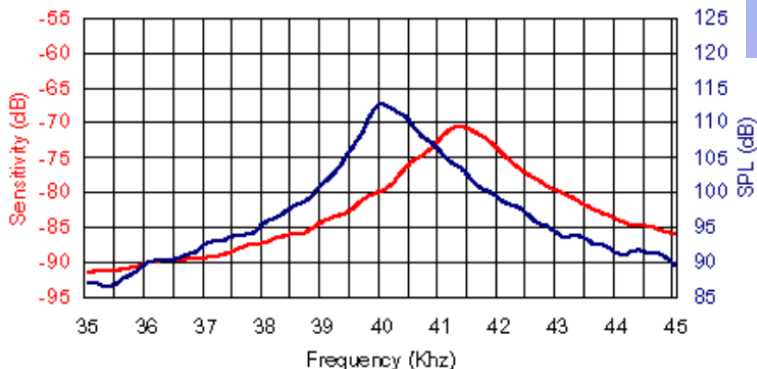
Rysunek: Directivity pattern of a piezoelectric transducer MA40B8R produced by Murata

Applications

- parking sensor,
- alarm protection,
- distance measurement,
- level of liquid.

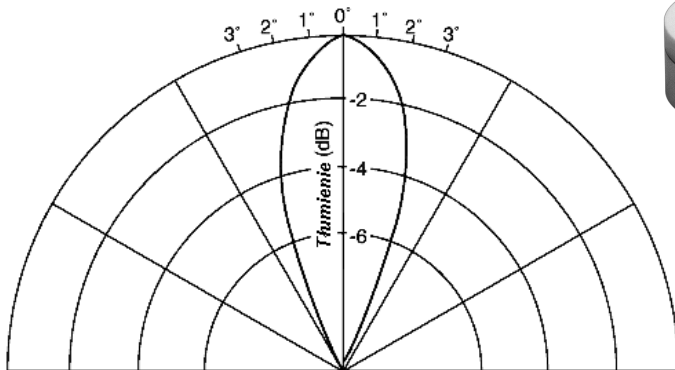


Ultrasonic transducers (transmitter/receiver)



Rysunek: Frequency diagram of a piezoelectric transducer 40KPT18A produced by Murata

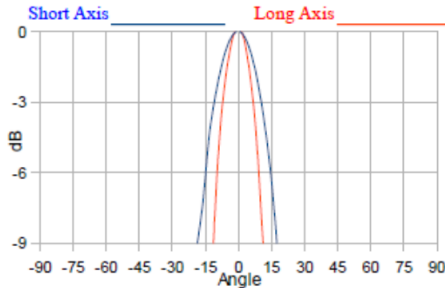
Ultrasonic transducer



Rysunek: Directivity pattern of a piezoelectric transducer MA80BA1 produced Murata

Ultrasonic transducers (transmitter/receiver)

Beam Angle: @48KHz



Rysunek: Directivity pattern of a piezoelectric transducer series 9000 produced by SensComp

This kind of directivity pattern is useful for cars. It can be detected what is above a pavement. Curbs don't cause alarm.

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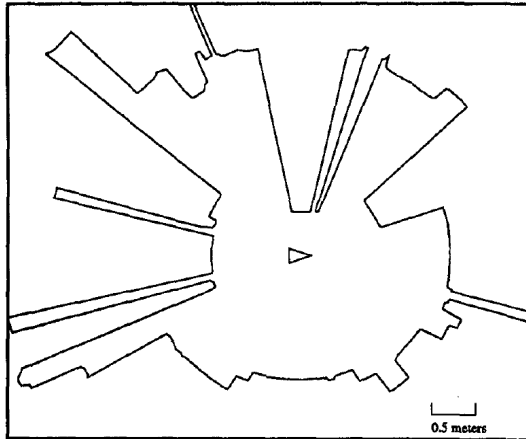
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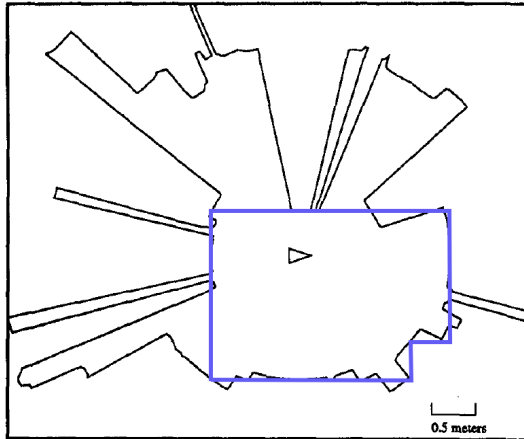
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How a room does it look like?



J. J. Leonard and H. F. Durrant-Whyte. Application of Multi-Target Tracking to Sonar-based Mobile Robot Navigation, *Proc. of the 29th Conference on Decision and Control*, Honolulu, Hawaii December 1990, pp 3118 – 3123

How a room does it look like?



J. J. Leonard and H. F. Durrant-Whyte. Application of Multi-Target Tracking to Sonar-based Mobile Robot Navigation, *Proc. of the 29th Conference on Decision and Control*, Honolulu, Hawaii December 1990, pp 3118 – 3123

Why is it so?

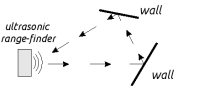
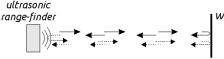
If a wavelength is much bigger than irregularities on a surface then a signal is reflected like a light beam. Angle of incidence is equal to angle of reflection.

The wavelength of most often used ultrasonic signals in range finders:

40kHz ~ 8,6mm (piezoelectric)

50kHz ~ 6,7mm (electrostatic)

The basic errors and their reasons

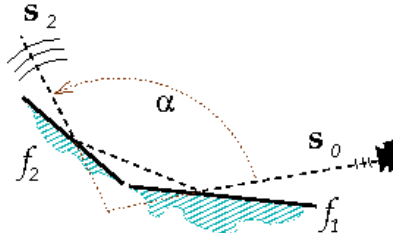
Schema of measurement performance	Results of measurement
 ultrasonic range-finder acoustic axis pole	 ultrasonic range-finder
 ultrasonic range-finder acoustic axis wall	 ultrasonic range-finder
 ultrasonic range-finder wall wall	 ultrasonic range-finder
 ultrasonic range-finder wall	 ultrasonic range-finder

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Reflection against two walls

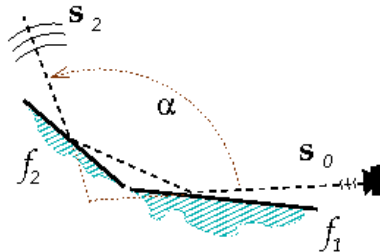
$$\angle(\vec{s}_0, \vec{s}_2) = 2\angle(\vec{f}_1, \vec{f}_2)$$



For two mirrors the angle between an emitted and arrival one do not depend on mirrors orientation in relation to a source of a signal.

Reflection against two walls

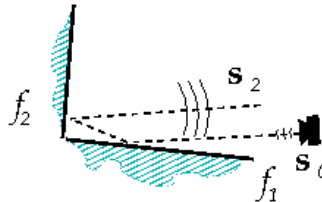
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For two mirrors the angle between an emitted and arrival one do not depend on mirrors orientation in relation to a source of a signal.

Retroreflector

$$\angle(\vec{f}_1, \vec{f}_2) = \frac{\pi}{2} \longrightarrow \angle(\vec{s}_0, \vec{s}_2) = 2\angle(\vec{f}_1, \vec{f}_2) = \pi$$



When mirrors are joined at a right angle, an emitted signal is returned back exactly to the source of the signal.

Retroreflectors



Ultrasonic sensing

Reflection angle for a set of mirrors

Echo threshold detection method

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Retroreflectors



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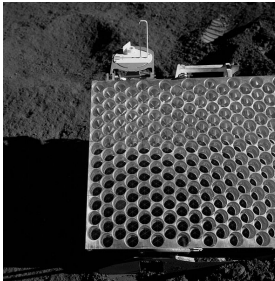
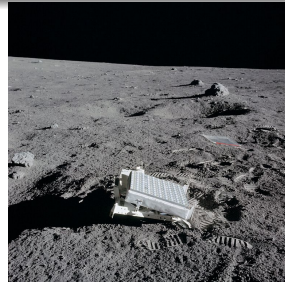
How it is exploited?

What a robot can see by using ultrasonic range finders

Retroreflectors



Retroreflectors on the Moon



Retroreflectors were left on the Moon by Apollos 11, 14 and 15. They were used for measuring the distance to the Moon.

Retroreflectors on the Moon



The ongoing Lunar Laser Ranging Experiment measures the distance between Earth and the Moon using laser ranging. Lasers on Earth are aimed at retroreflectors planted on the Moon during the Apollo program (11, 14, and 15) and the two Lunokhod missions. The time for the reflected light to return is measured. At the Moon's surface, the beam is about 6.5 kilometers wide and scientists liken the task of aiming the beam to using a rifle to hit a moving dime 3 kilometers away. This is one of the most precise distance measurements ever made, and is equivalent in accuracy to determining the distance between Los Angeles and New York to 0.25 mm.



The diameter of a dime is almost a centimeter.

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Stealth Technology



F-15 Eagle

Ultrasonic sensing

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Stealth Technology



F-22 Raptor

Ultrasonic sensing

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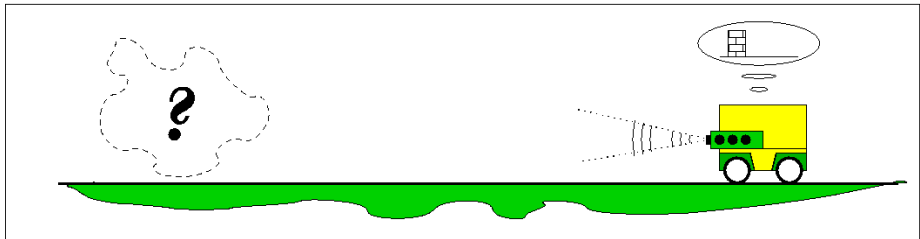
F-117 Night Hawk

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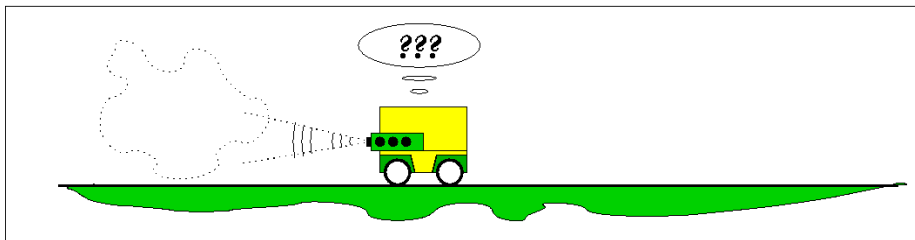
Examples of echos

(1)



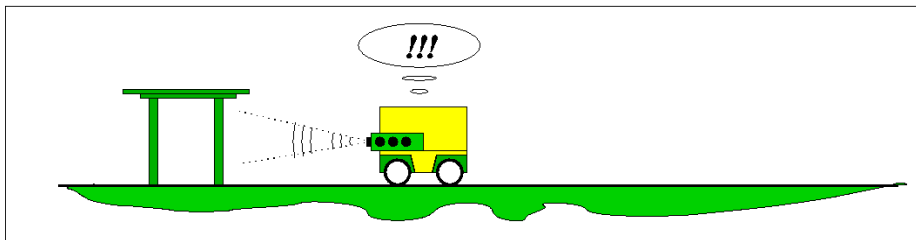
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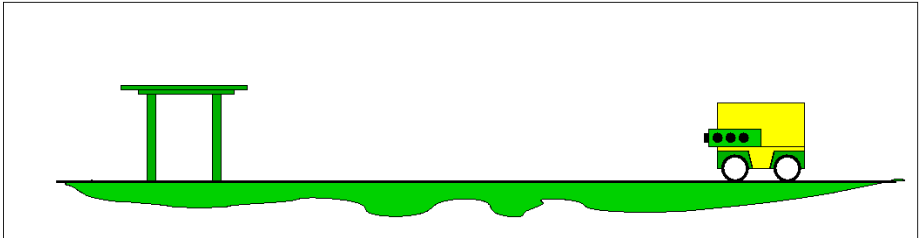
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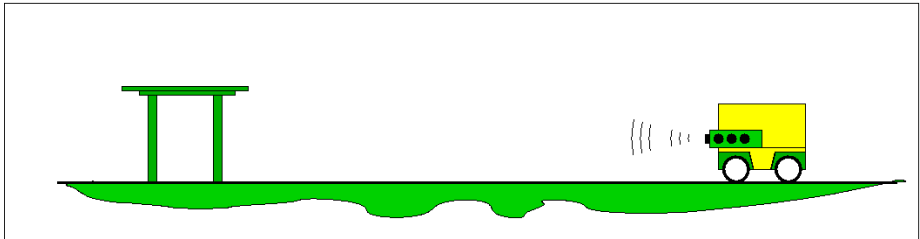
How does it work?

(1)



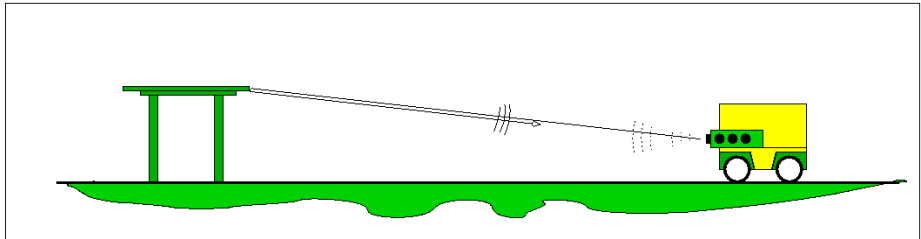
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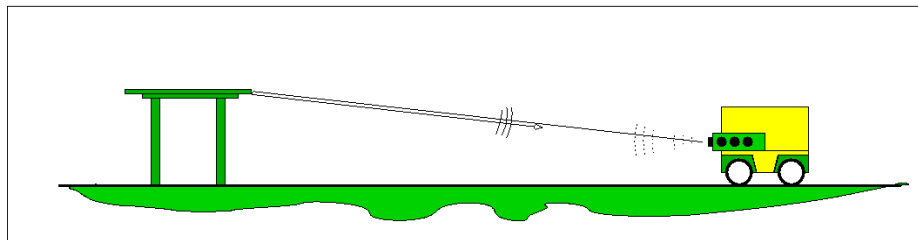
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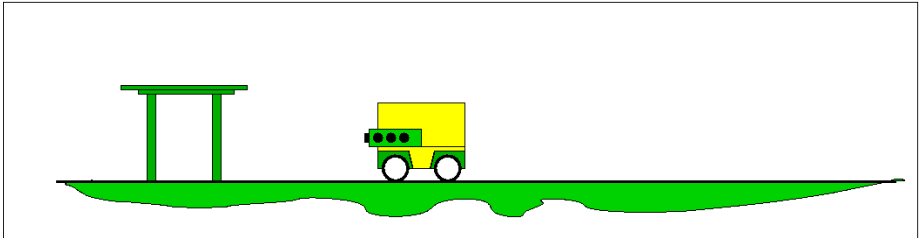
(1)



An edge of a table produces the same echo like a pole.

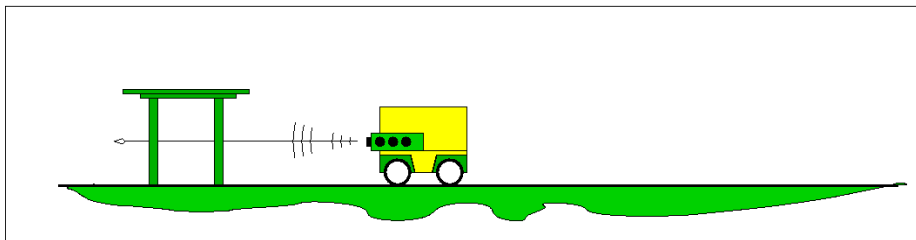
How does it work?

(1)



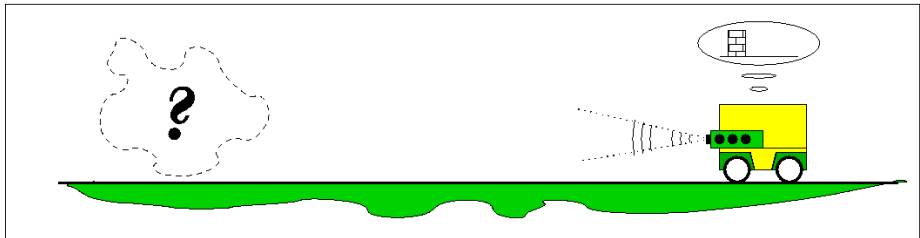
How does it work?

(1)



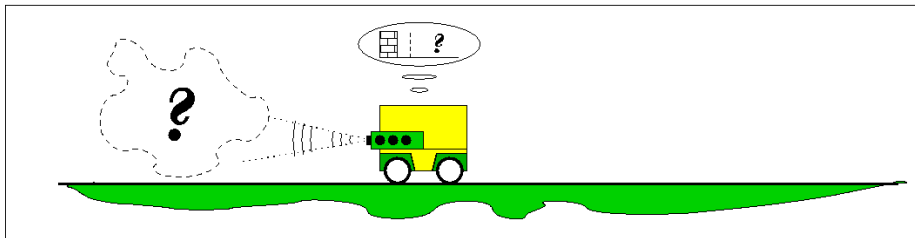
An example of a double reflection

(2)



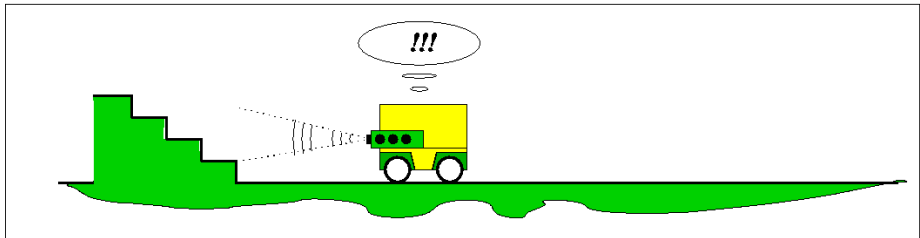
An example of a double reflection

(2)



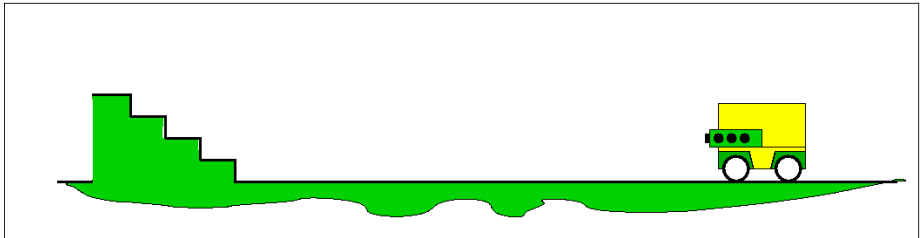
An example of a double reflection

(2)



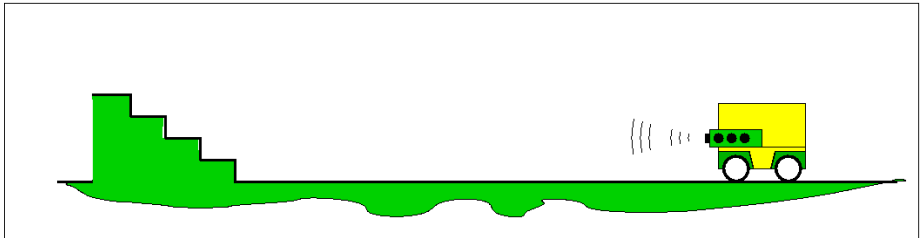
How does it work?

(2)



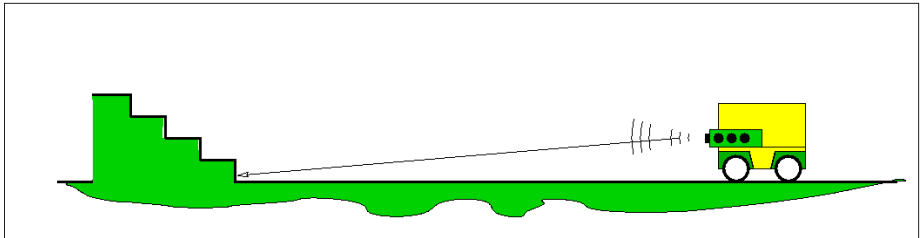
How does it work?

(2)



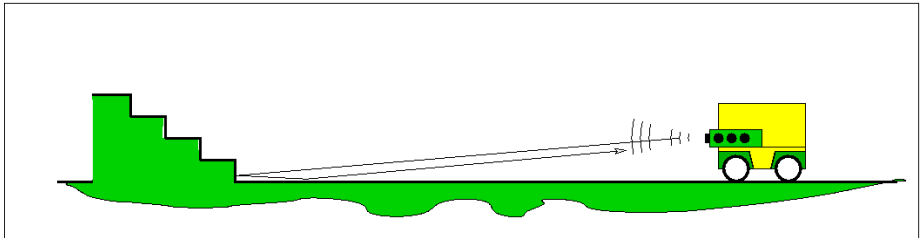
How does it work?

(2)



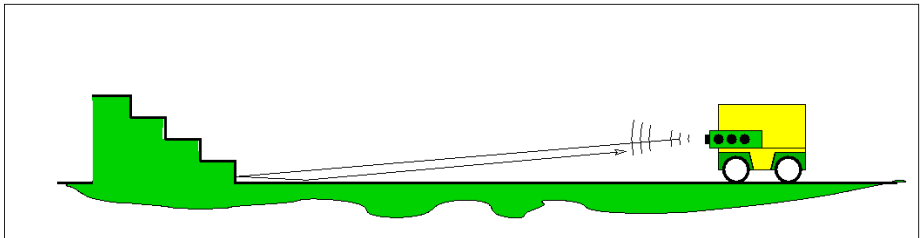
How does it work?

(2)



How does it work?

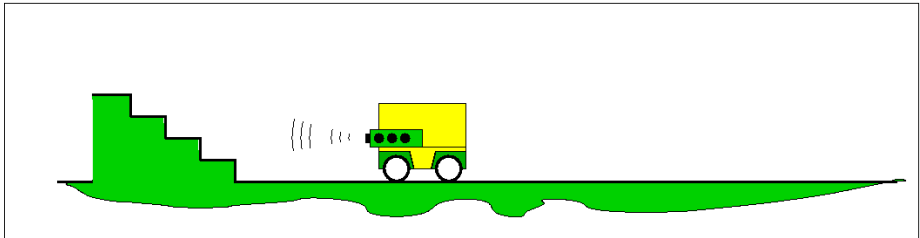
(2)



Stairs can be a source of double reflection as well as a corner of a room.

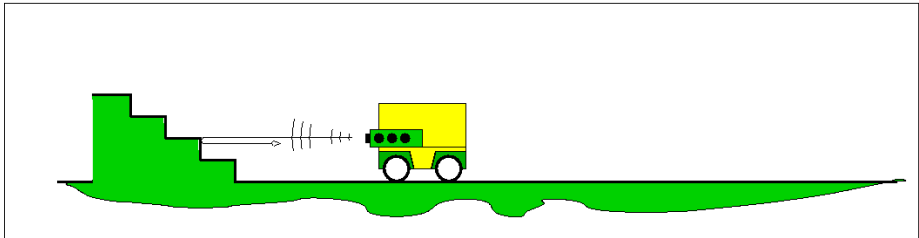
How does it work?

(2)



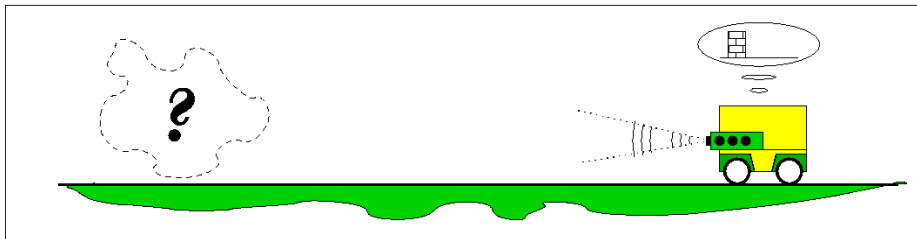
How does it work?

(2)



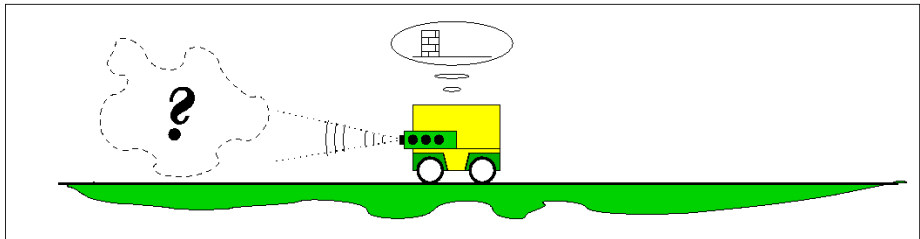
Examples of echos

(3)



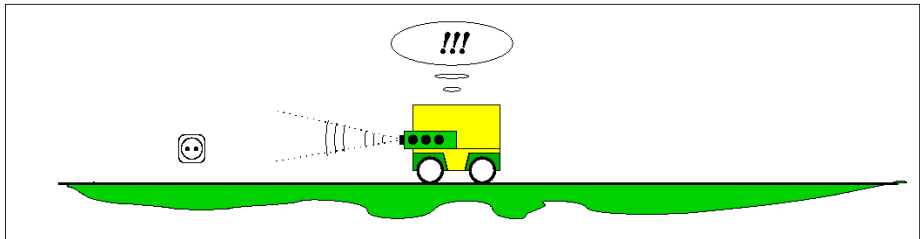
Examples of echos

(3)



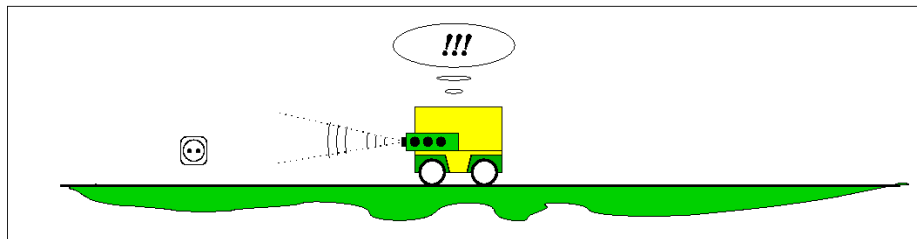
Examples of echos

(3)



Examples of echos

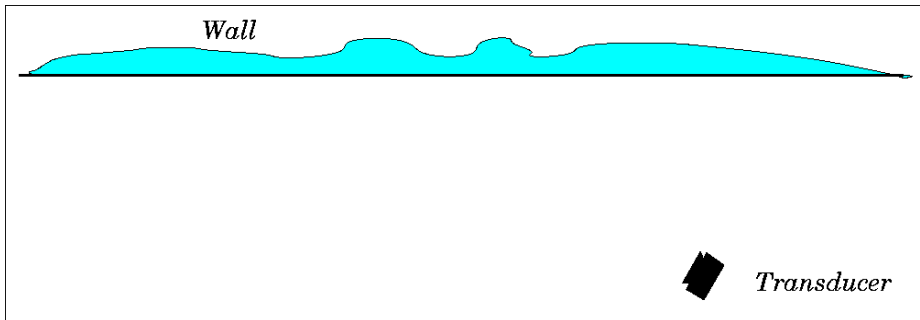
(3)



An electric socket can be a source of the same echo like a point-like object.

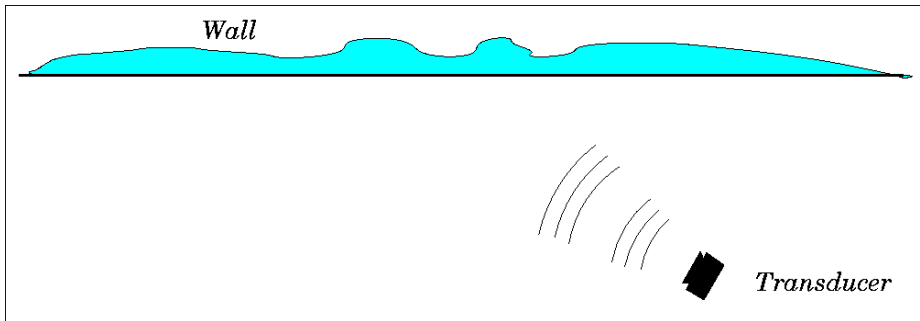
How does it work?

(3)



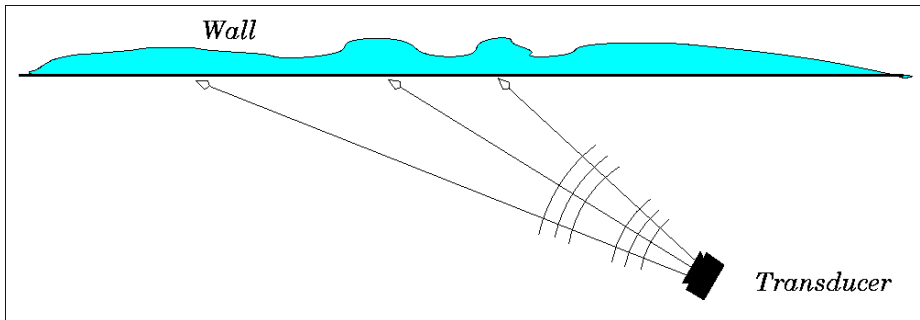
How does it work?

(3)



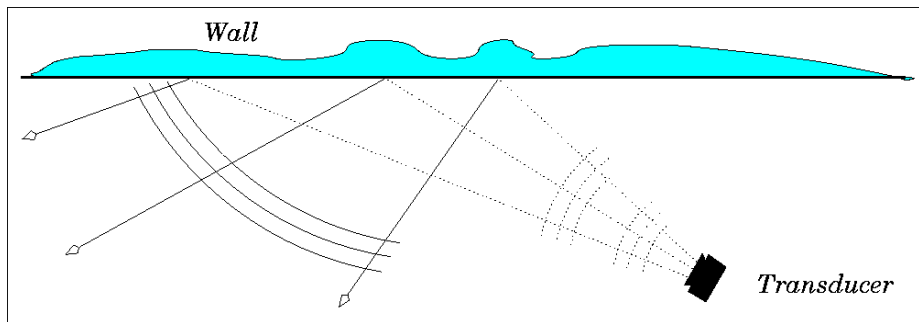
How does it work?

(3)



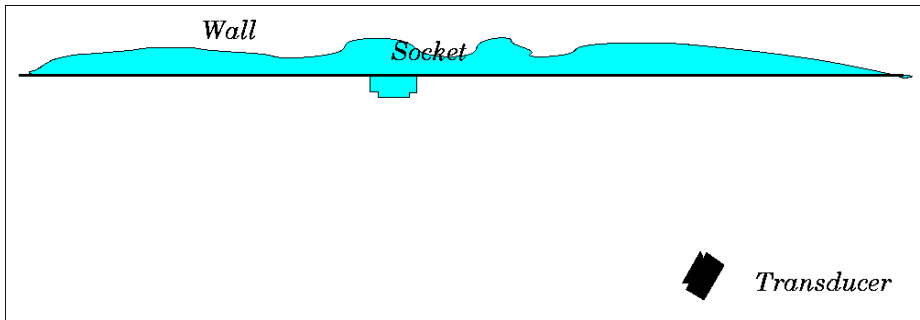
How does it work?

(3)



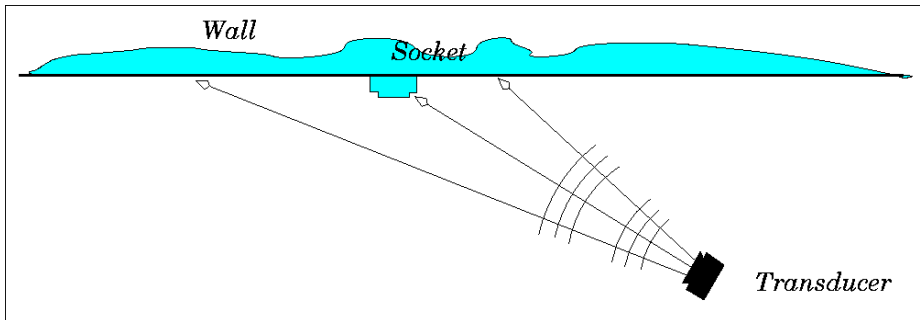
How does it work?

(3)



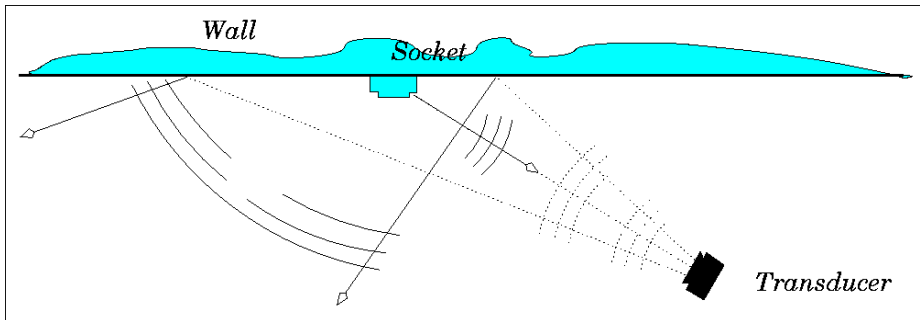
How does it work?

(3)



How does it work?

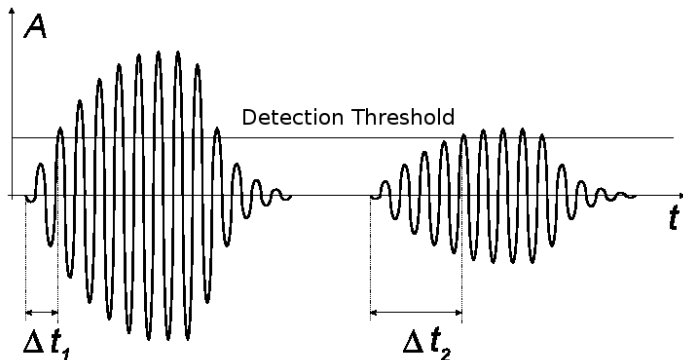
(3)



Plan prezentacji

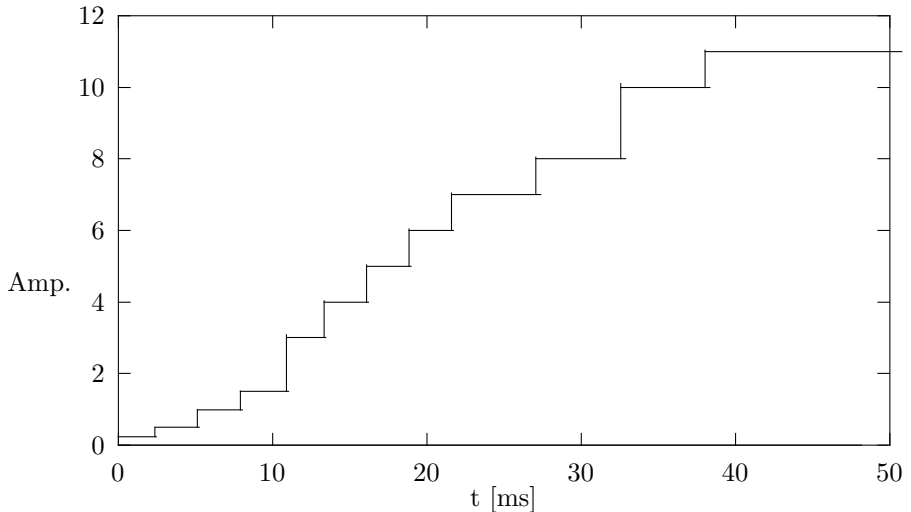
- 1 Ultrasonic sensing
 - Electrostatic transducers
 - Piezoelectric transducers
 - Features of ultrasonic sensing
- 2 Reflection angle for a set of mirrors
 - Interpretation of Equations
 - How it is exploited?
 - What a robot can see by using ultrasonic range finders
- 3 **Echo threshold detection method**
 - **Principles of operations**
 - Measurements—main features

Signal threshold detection



Rysunek: Delay of echo detection

Example of signal amplification



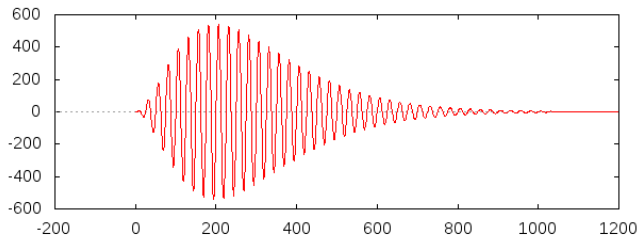
Threshold detection method

Advantages:

- simplicity,
- easy for technical application.

Disadvantages:

- the final result depends on the signal strength,
- cannot distinguish overlapping echos,
- sensitivity to a noise.



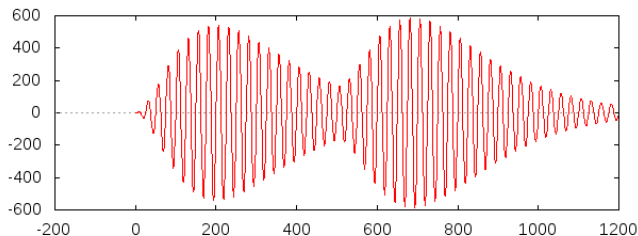
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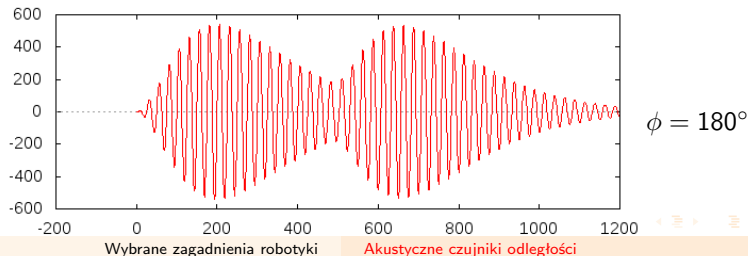
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Plan prezentacji

- 1 Ultrasonic sensing
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 - **Measurements—main features**

Received signal



Received signal



Received signal



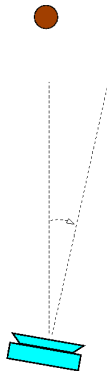
Received signal



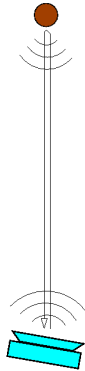
Received signal



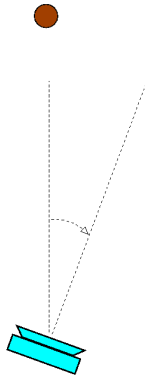
Received signal



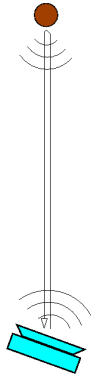
Received signal



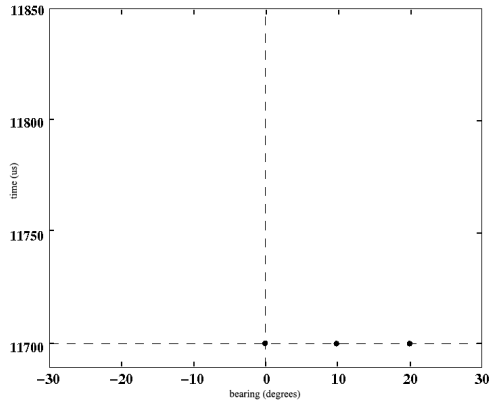
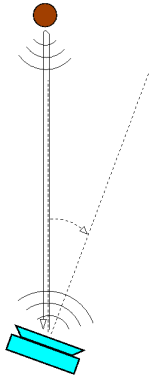
Received signal



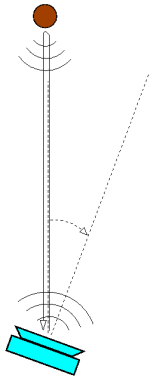
Received signal



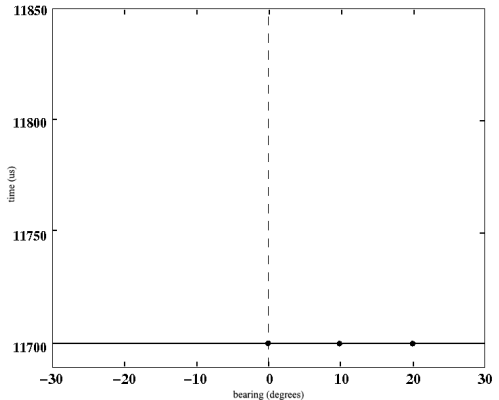
Received signal



Received signal



Measurement data were taken from:

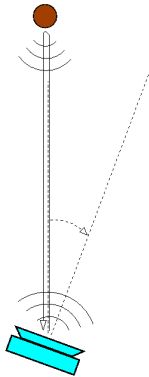


R. Kuc.

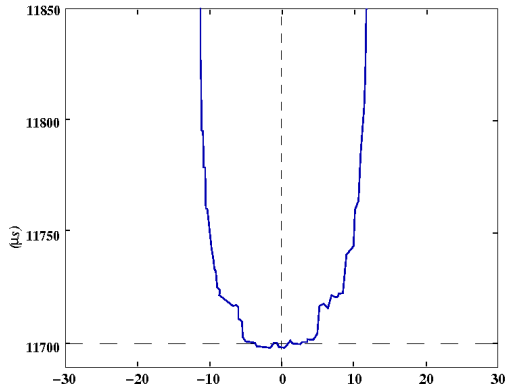
Forward model for sonar maps produced with the polaroid ranging module.

IEEE Trans. Robot. Automat., 19(2), April 2003.

Received signal



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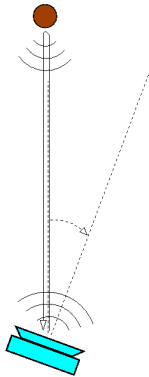


R. Kuc.

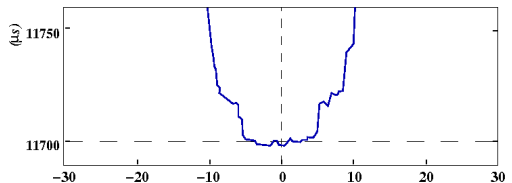
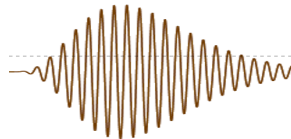
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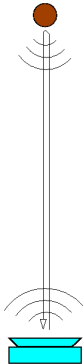


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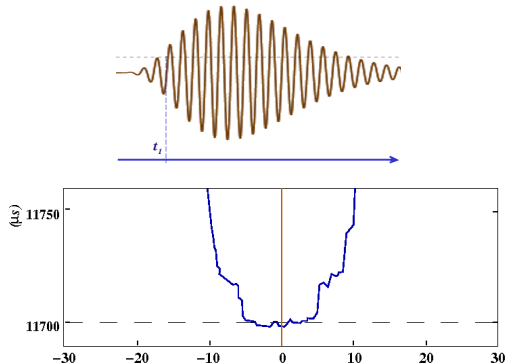
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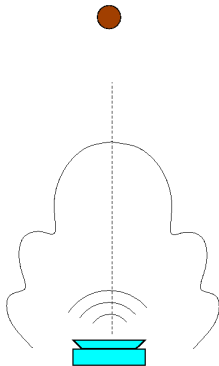


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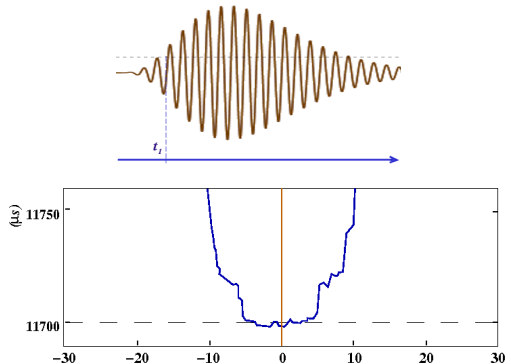
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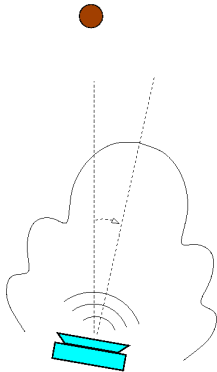


R. Kuc.

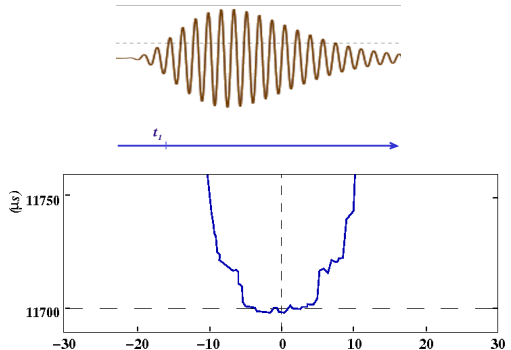
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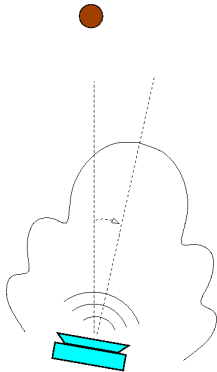


R. Kuc.

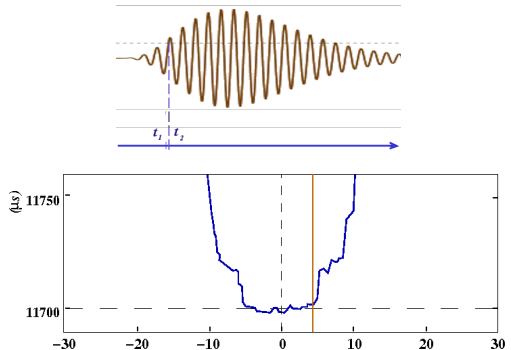
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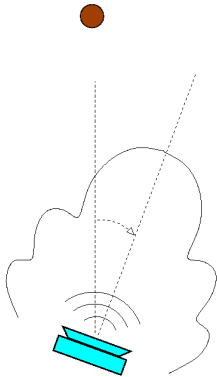


R. Kuc.

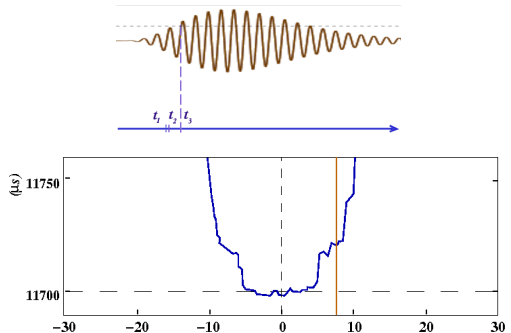
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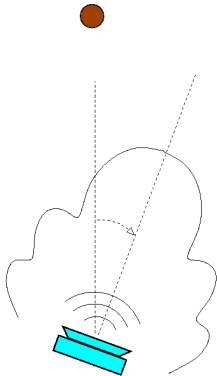


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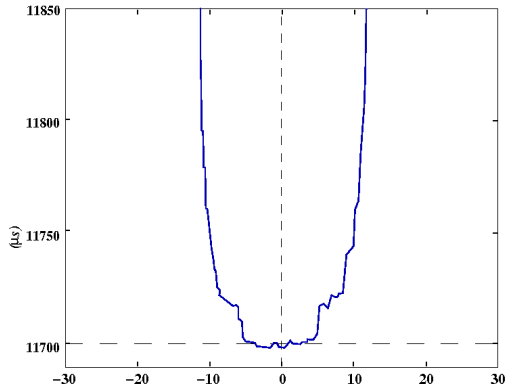
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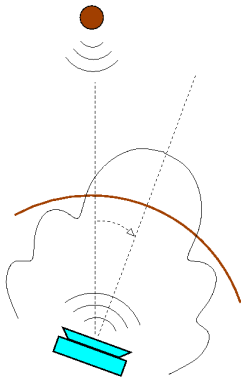


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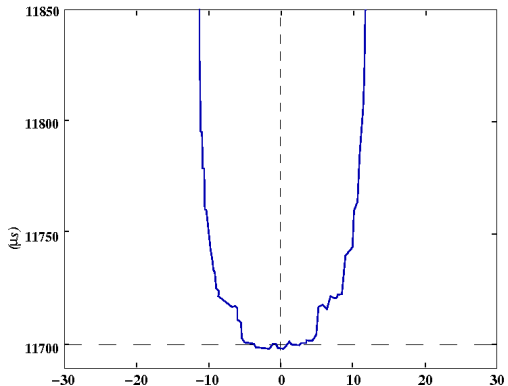
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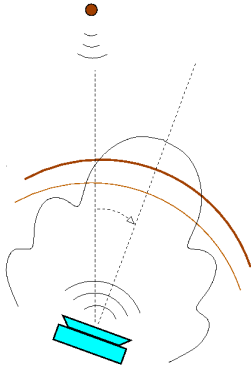


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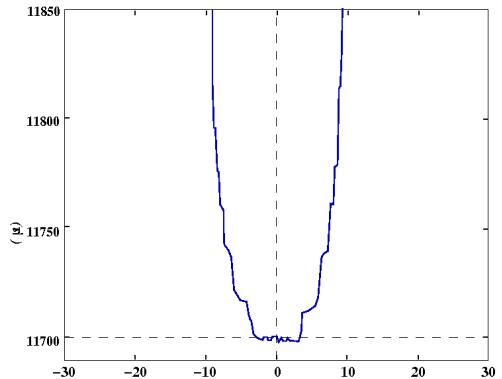
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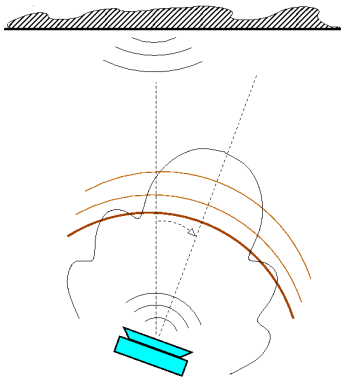


R. Kuc.

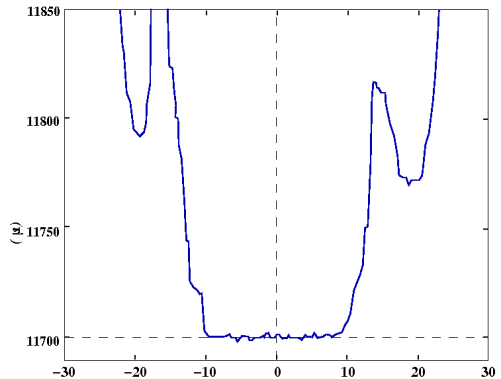
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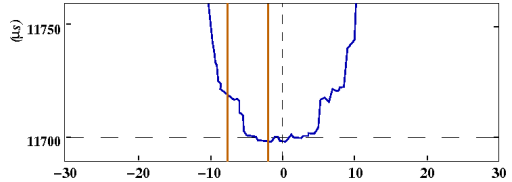
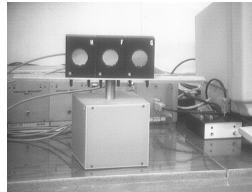
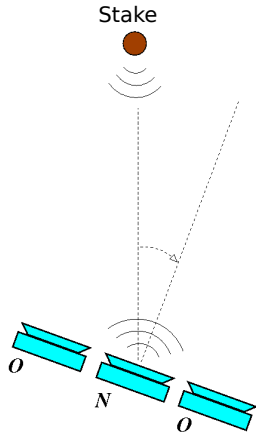


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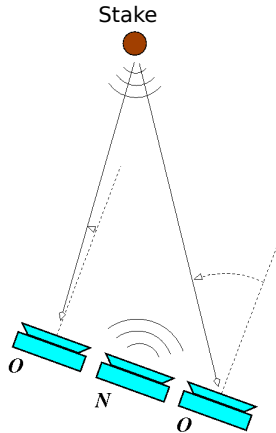
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IEEE Trans. Robot. Automat., 19(2), April 2003.

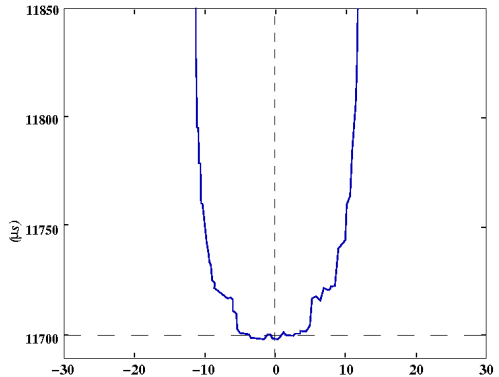
Stereo measurement



Stereo measurement



Measurement data were taken from:

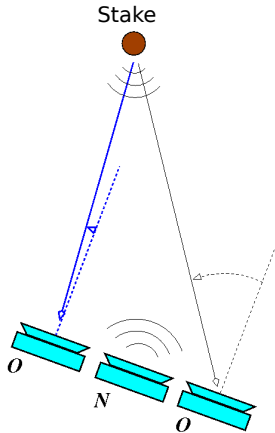


R. Kuc.

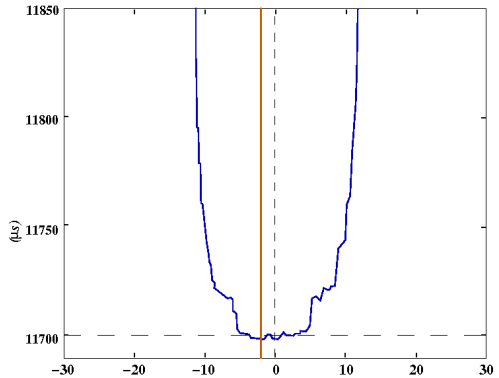
Forward model for sonar maps produced with the polaroid ranging module.

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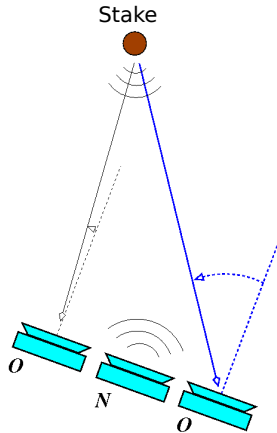


R. Kuc.

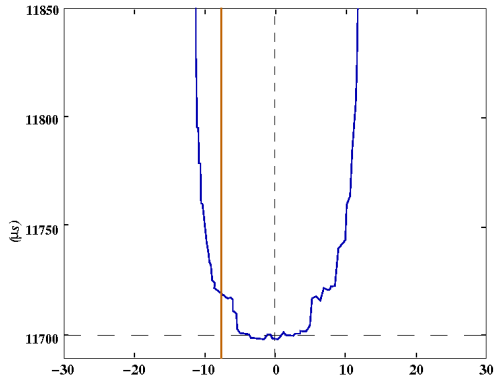
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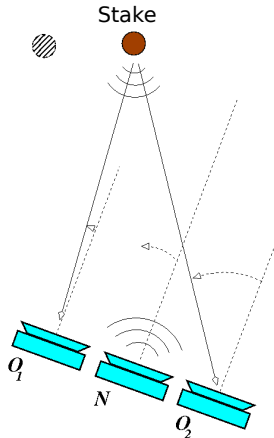


R. Kuc.

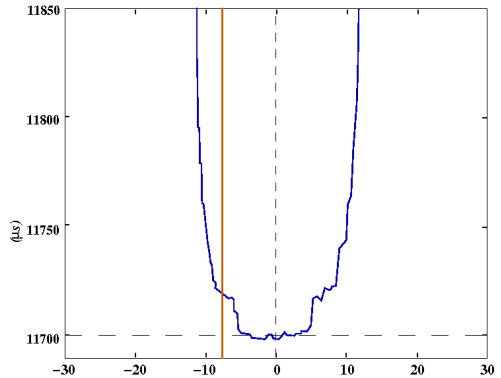
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Measurement data were taken from:

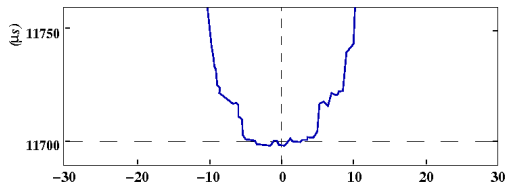
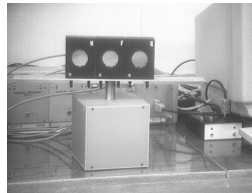
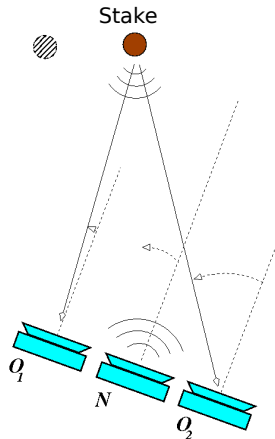


R. Kuc.

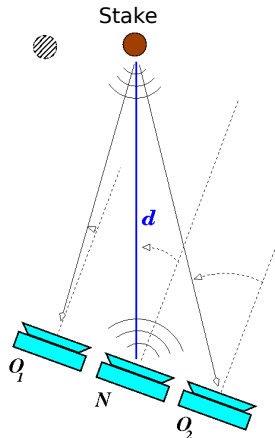
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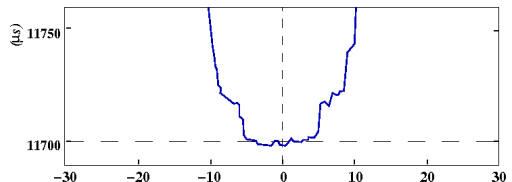
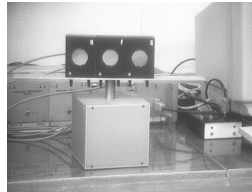
Stereo measurement—numerical example



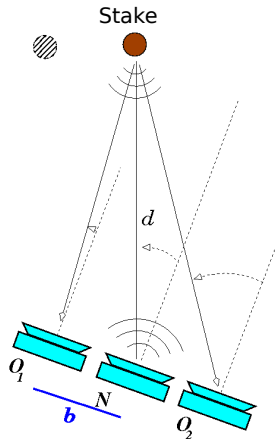
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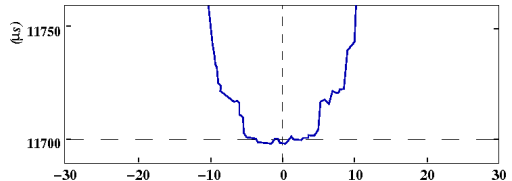
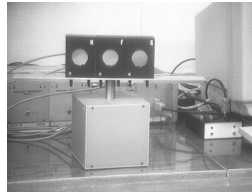
$$d = 200\text{cm}$$



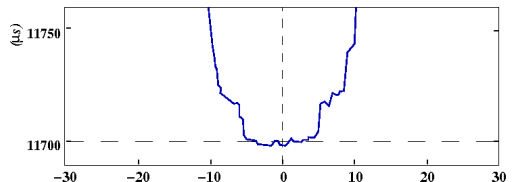
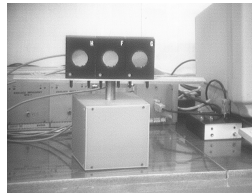
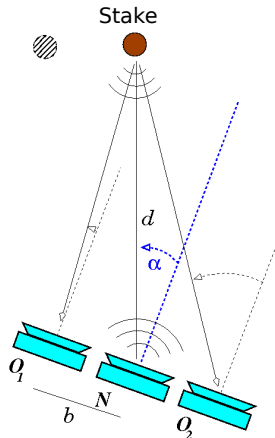
Stereo measurement—numerical example



$$d = 200\text{cm}, b = 7\text{cm}$$

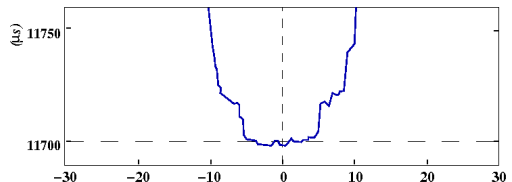
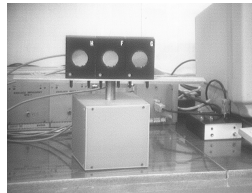
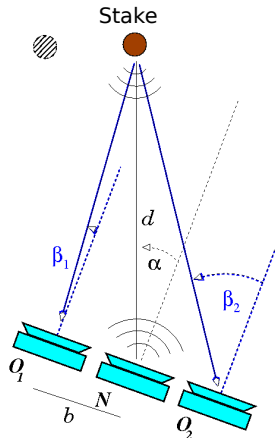


Stereo measurement—numerical example



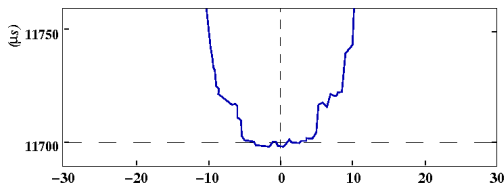
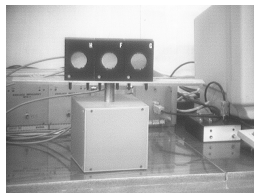
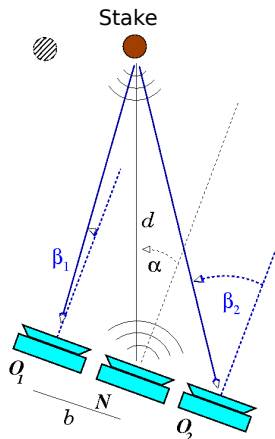
$$d = 200\text{cm}, b = 7\text{cm}, \alpha = 5^\circ$$

Stereo measurement—numerical example



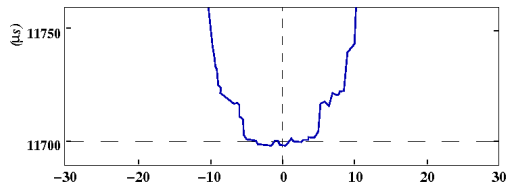
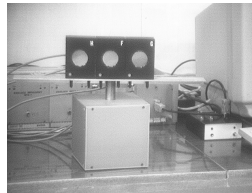
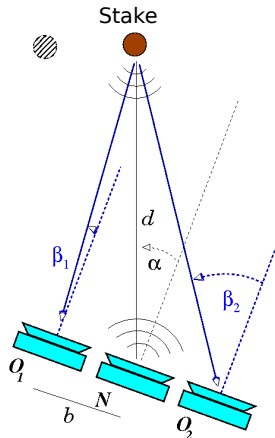
$$d = 200\text{cm}, b = 7\text{cm}, \alpha = 5^\circ \rightarrow \beta_1 = 3,0^\circ, \beta_2 = 7,0^\circ$$

Stereo measurement—numerical example



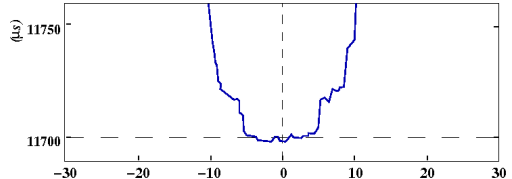
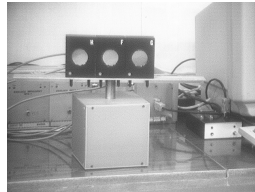
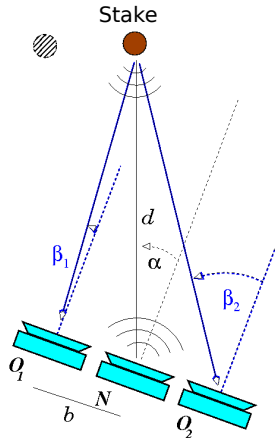
$$d = 200\text{cm}, b = 7\text{cm}, \alpha = 5^\circ \longrightarrow \beta_1 = 3,0^\circ, \beta_2 = 7,0^\circ \longrightarrow \Delta\beta = 4,0^\circ$$

Stereo measurement—numerical example



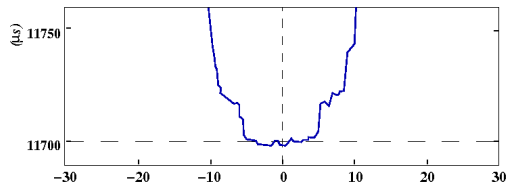
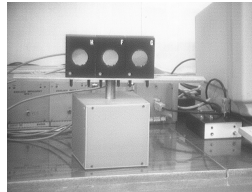
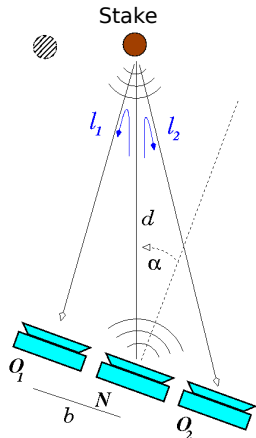
$$d = 100\text{cm}, b = 7\text{cm}, \alpha = 5^\circ \rightarrow \beta_1 = 1,0^\circ, \beta_2 = 9,0^\circ \rightarrow \Delta\beta = 8,0^\circ$$

Stereo measurement—numerical example



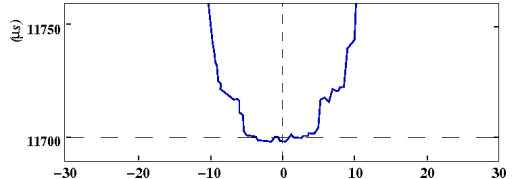
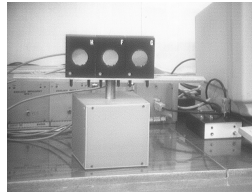
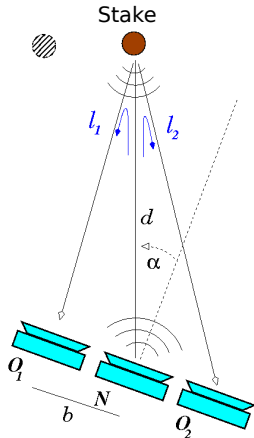
$$d = 300\text{cm}, b = 7\text{cm}, \alpha = 5^\circ \rightarrow \beta_1 = 3,7^\circ, \beta_2 = 6,3^\circ \rightarrow \Delta\beta = 2,6^\circ$$

Stereo measurement—numerical example



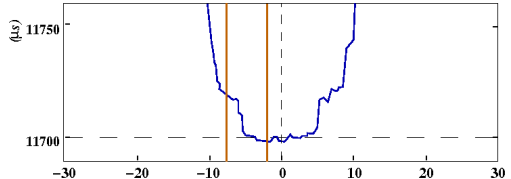
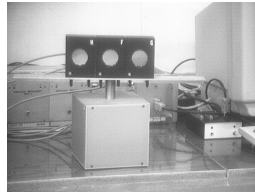
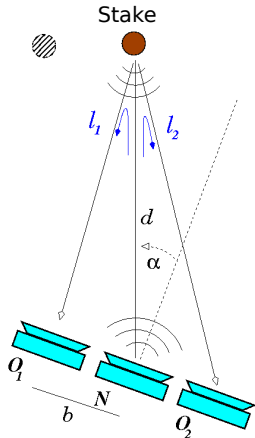
$$d = 200\text{cm}, b = 7\text{cm}, \alpha = 5^\circ$$

Stereo measurement—numerical example



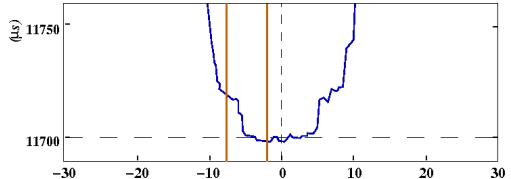
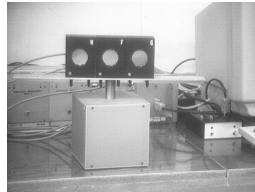
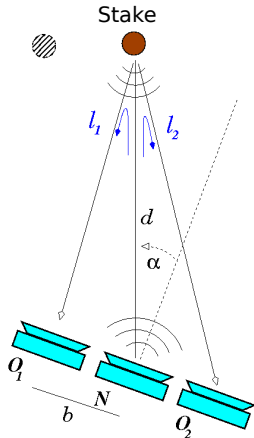
$$d = 200\text{cm}, b = 7\text{cm}, \alpha = 5^\circ \rightarrow l_1 = 399,51\text{cm}, l_2 = 400,73\text{cm}$$

Stereo measurement—numerical example



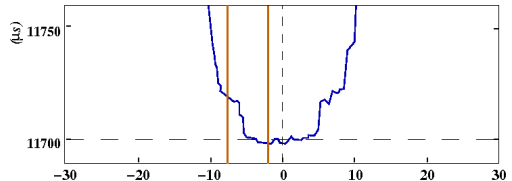
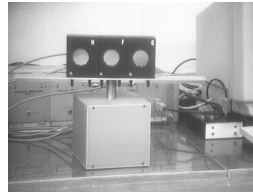
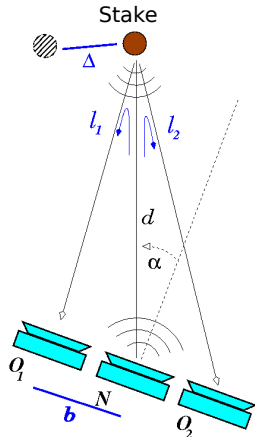
$$d = 200\text{cm}, b = 7\text{cm}, \alpha = 5^\circ \longrightarrow l_1 = 399,51\text{cm}, l_2 = 400,73\text{cm}$$

Stereo measurement—numerical example



$$d = 200\text{cm}, b = 7\text{cm}, \alpha = 5^\circ \longrightarrow l_1 = 399,51\text{cm}, l_2 = 400,73\text{cm} + 0,7\text{cm}$$

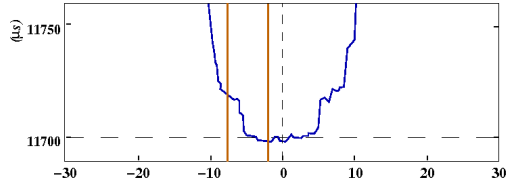
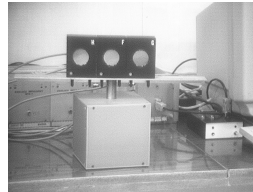
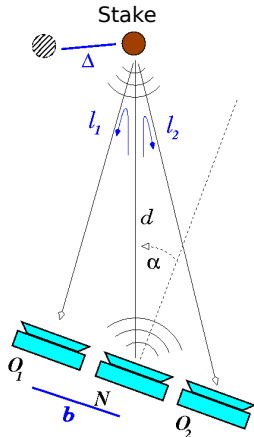
Stereo measurement—numerical example



$$d = 200\text{cm}, b = 7\text{cm}, \alpha = 5^\circ \longrightarrow l_1 = 399,51\text{cm}, l_2 = 400,73\text{cm} + 0,7\text{cm}$$

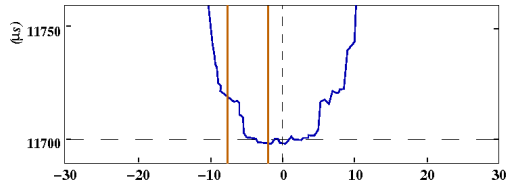
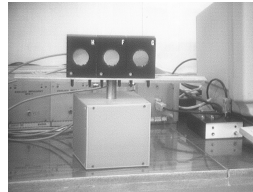
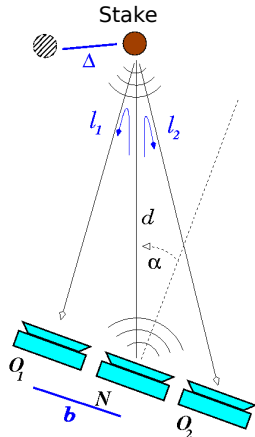
$$d = 200,18\text{cm} \quad \delta = 1,4^\circ \quad \Delta = 7,1\text{cm}$$

Stereo measurement—numerical example



$$d = 300\text{cm}, b = 7\text{cm}, \alpha = 5^\circ \rightarrow l_1 = 599,47\text{cm}, l_2 = 600,69\text{cm} + 0,7\text{cm}$$

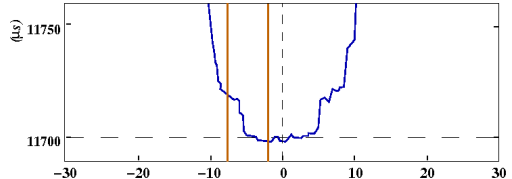
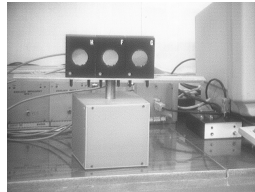
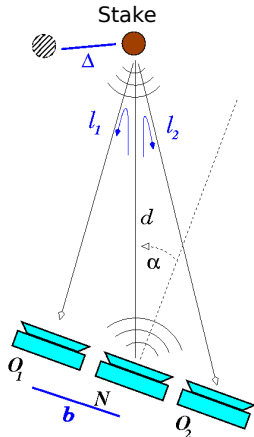
Stereo measurement—numerical example



$$d = 300\text{cm}, b = 7\text{cm}, \alpha = 5^\circ \longrightarrow l_1 = 599,47\text{cm}, l_2 = 600,69\text{cm} + 0,7\text{cm}$$

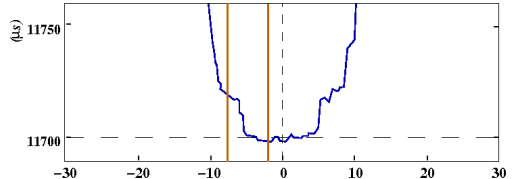
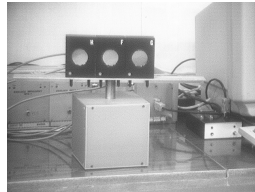
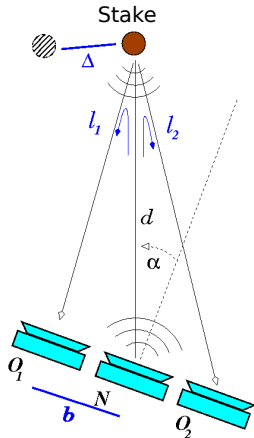
$$d = 300,18\text{cm} \quad \delta = 1,4^\circ \quad \Delta = 10,6\text{cm}$$

Stereo measurement—numerical example



$$d = 100\text{cm}, b = 7\text{cm}, \alpha = 5^\circ \longrightarrow l_1 = 99,63\text{cm}, l_2 = 100,85\text{cm} + 0,7\text{cm}$$

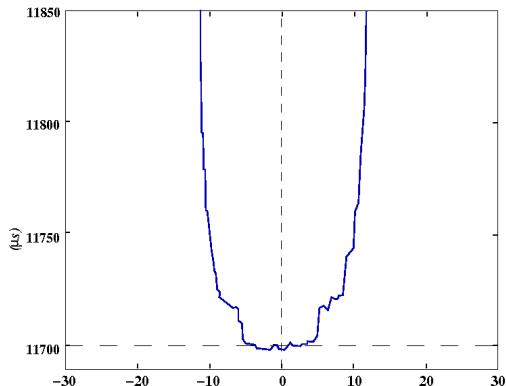
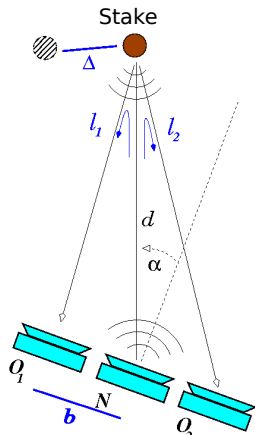
Stereo measurement—numerical example



$$d = 100\text{cm}, b = 7\text{cm}, \alpha = 5^\circ \longrightarrow l_1 = 99,63\text{cm}, l_2 = 100,85\text{cm} + 0,7\text{cm}$$

$$d = 100,18\text{cm} \quad \delta = 1,4^\circ \quad \Delta = 3,6\text{cm}$$

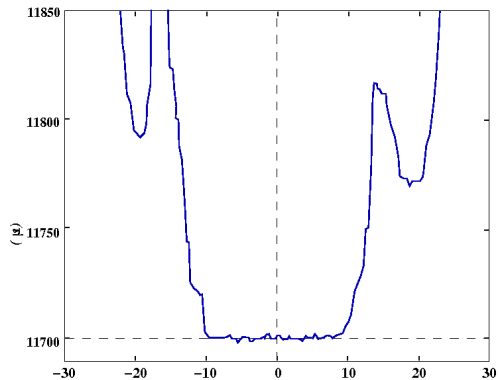
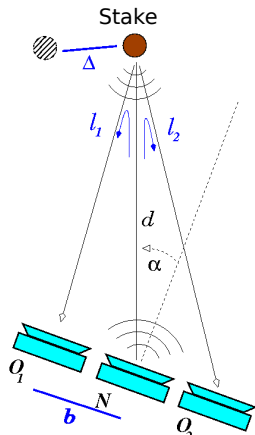
Stereo measurement—numerical example



$$d = 100\text{cm}, b = 7\text{cm}, \alpha = 5^\circ \rightarrow l_1 = 99,63\text{cm}, l_2 = 100,85\text{cm} + 0,7\text{cm}$$

$$d = 100,18\text{cm} \quad \delta = 1,4^\circ \quad \Delta = 3,6\text{cm}$$

Stereo measurement—numerical example



$$d = 100\text{cm}, b = 7\text{cm}, \alpha = 5^\circ \rightarrow l_1 = 99,63\text{cm}, l_2 = 100,85\text{cm} + 0,7\text{cm}$$

$$d = 100,18\text{cm} \quad \delta = 1,4^\circ \quad \Delta = 3,6\text{cm}$$

Koniec prezentacji
Dziękuję za uwagę