Formalizm lagranżowski

Algorytm

1° $q, \dot{q} \in R^n$, $t \in [t_0, t_1]$

2° $L(q,\dot{q}) = K(q,\dot{q}) - V(q)$

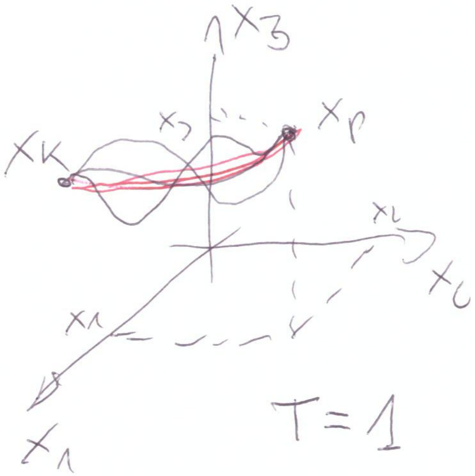
3° ZND $I(q(\cdot)) = \int_{t_0}^{t_1} L(q,\dot{q})\, dt \to extr$

4° REL $\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = 0$ $= F$

Postać energii kin. $K(q,\dot{q}) = \frac{1}{2}\dot{q}^T Q(q)\dot{q}$

$+R\dot{q}$ ← tarcie

Postać równań $\boxed{Q(q)\ddot{q} + C(q,\dot{q})\dot{q} + D(q) = F}$



$T=1$

Integrator Brockette

$$\begin{cases}\dot{x}_1 = u_1\\ \dot{x}_2 = u_2\\ \dot{x}_3 = x_1u_2 - x_2u_1\end{cases} \Rightarrow \underbrace{\dot{x}_3 - x_1u_2 - x_2u_1}_{G(x)} = 0$$

$$\int_0^1 \underbrace{\left(u_1^2 + u_2^2\right)}_{L} dt \to \min$$

$$\mathcal{L} = L + \lambda(t)K = u_1^2 + u_2^2 + \lambda(t)\left(\dot{x}_3 - x_1u_2 - x_2u_1\right)$$

$$\frac{d}{dt}\frac{\partial L(q,\dot q)}{\partial \dot q} - \frac{\partial L(q\dot q)}{\partial q} = 0$$

$$\frac{1}{2}\sigma^T m \sigma \qquad \frac{\partial^2 L}{\partial \dot q^2}\ddot q + \frac{\partial^2 L}{\partial \dot q \partial q}\dot q - \frac{\partial L}{\partial q} = 0\ F$$

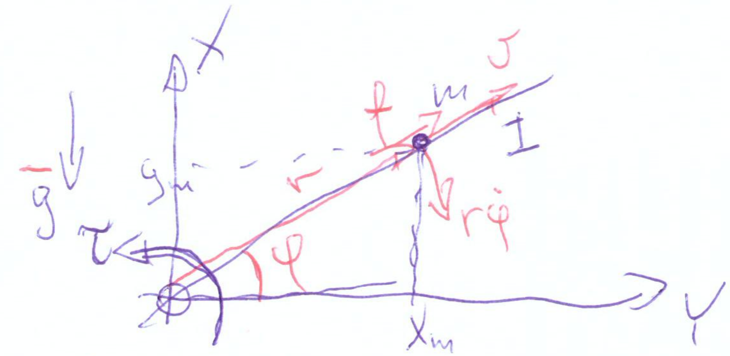
$\frac{1}{2} m \sigma^2$ Q C D

$$\cancel{Q\ddot q + C} \quad Q(q)\ddot q + C(q\dot q)\dot q + D(q) = F$$

$$K(q\dot q) = \dot q^T Q \dot q \qquad Q(q)\ddot q = F - C(q\dot q)\dot q - D(q)$$

$$m\ a = F$$

$$Q^{-1}\ \ddot q = Q^{-1}(\ \ \ )$$



$q=\begin{pmatrix} x \\ y \end{pmatrix}$ $\dot{q}=\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix}$ $K_m = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2)$

$q=\begin{pmatrix} r \\ \varphi \end{pmatrix}$ $\dot{q}=\begin{pmatrix} \dot{r} \\ \dot{\varphi} \end{pmatrix}$ $L = K - V$

$$K = \frac{1}{2} I \dot{\varphi}^2 + \frac{1}{2} m v^2$$

$$\frac{1}{2} m \dot{r}^2 + \frac{1}{2} m r^2 \dot{\varphi}^2$$

$\bar{g} = \begin{pmatrix} 0 \\ -g \end{pmatrix}$ $\vec{r} = \begin{pmatrix} x_m \\ y_m \end{pmatrix}$

$y_m = r \sin\varphi$

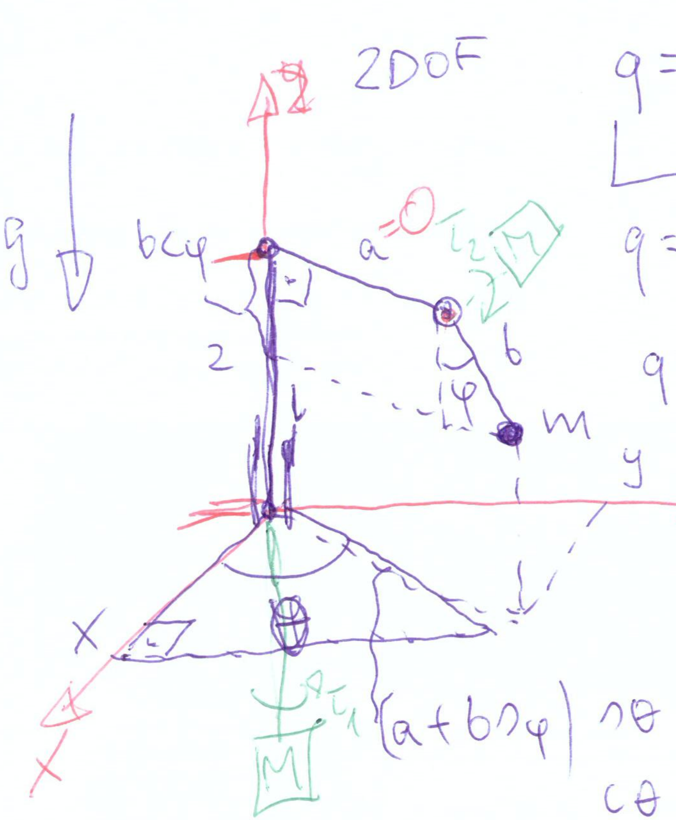
$$V = -m(\vec{g}, \vec{r}) = \qquad mg\underline{\underline{h}}$$

$$= -m(0, -g)\begin{pmatrix} x_m \\ y_m \end{pmatrix} = mg y_m = mgr\sin\varphi$$

$$L = \frac{1}{2} I \dot{\varphi}^2 + \frac{1}{2} m r^2 \dot{\varphi}^2 + \frac{1}{2} m \dot{r}^2 - mgr\sin\varphi$$

$$= \frac{1}{2}\underbrace{(I + m r^2)}\dot{\varphi}^2 + \frac{1}{2} m \dot{r}^2 - mgr\sin\varphi$$

rovnice $\begin{cases} \text{cos } 1 & = 0 \cdot f \\ \text{cos } 2 & = 0 \cdot \tau \end{cases}$ vidle skript



$$\boxed{q=\begin{pmatrix}x\\y\\z\end{pmatrix}}$$

$$q=\begin{pmatrix}x\\y\end{pmatrix}$$

$$q=\begin{pmatrix}y\\z\end{pmatrix}$$

$$K=\frac{1}{2}m\left(\dot{x}^2+\dot{y}^2+\dot{z}^2\right) \qquad V=mgz \qquad c\varphi=\cos\varphi \quad s\varphi=\sin\varphi$$

ogr

$$q=\begin{pmatrix}\theta\\\varphi\end{pmatrix} \quad \dot{q}=\begin{pmatrix}\dot\theta\\\dot\varphi\end{pmatrix} \qquad b=0$$

$$\boxed{\begin{cases} x_m=(a+b\,s\varphi)\,c\theta\\ y_m=(a+b\,s\varphi)\,s\theta\\ z_m=l-b\,c\varphi\end{cases}}$$

$$L=K-V$$

$$\begin{cases} \dot{x}_m=\\ \dot{y}_m=\\ \dot{z}_m=l+b\,s\varphi\,\dot\varphi\end{cases}$$

$$\begin{cases} \frac{d}{dt}\frac{\partial L}{\partial\dot\theta}-\underbrace{\frac{\partial L}{\theta}}_{=0}=0 & \frac{\partial L}{\partial\dot\theta}=\text{const}\\ \frac{d}{dt}\frac{\partial L}{\partial\dot\varphi}-\frac{\partial L}{\partial\varphi}=0\end{cases}$$

θ 1rów $\begin{cases}\text{coś}1=\text{const}\\ \text{coś}2=0\ \tau_2\end{cases}$ $\begin{cases}\frac{d}{dt}\text{coś}1=0\ \tau_1\\ \text{coś}2=\tau_2\end{cases}$

φ 2rów

$$K=\frac{1}{2}\dot{q}^T\underset{\hat{R}}{\underline{\underline{Q}}}\dot{q}=\frac{1}{2}Q\dot{q}^T\dot{q}$$

Lagranżian

$$L = \dot q^T Q \dot q$$

Postać równań

$$Q\ddot q + C\dot q = 0$$

$$\Downarrow$$

$$Q\ddot q = -C\dot q$$

Własności

$$\dot Q = C + C^T$$

$$\frac{\partial L}{\partial t} = \underbrace{\left(\frac{1}{2}\ddot q^T Q \dot q\right)^T + \frac{1}{2}\dot q^T Q \ddot q}_{\dot q^T \underbrace{(Q^T + Q)}_{2Q} \ddot q} + \frac{1}{2}\dot q^T \dot Q \dot q$$

$(\ )^T$ bo to liczby

$2Q$ — Q macierz symetryczna

$$\frac{\partial L}{\partial t} = \dot q^T Q \ddot q + \frac{1}{2}\dot q \dot Q \dot q = -\dot q^T C \dot q + \frac{1}{2}\dot q^T \dot Q \dot q =$$

$$= \frac{1}{2}\dot q^T(\dot Q - 2C)\dot q = \frac{1}{2}\dot q^T \underbrace{(C + C^T - 2C)}_{C^T - C}\dot q = \frac{1}{2}\dot q^T (C^T - C)\dot q$$

co to $C^T - C$? oznaczmy jako M

$$M = C^T - C \qquad M^T = (C^T - C)^T = C - C^T = -(C^T - C) = -M$$

$M^T = -M$ — macierz skośnie symetryczna

jej własności:

forma kwadratowa zbudowana na macierzy skośniesymetrycznej jest równa 0

więc $\boxed{\frac{\partial L}{\partial t} = \frac{1}{2}\dot q^T (C^T - C)\dot q = 0}$

Lagranżian jest stały na trajektoriach