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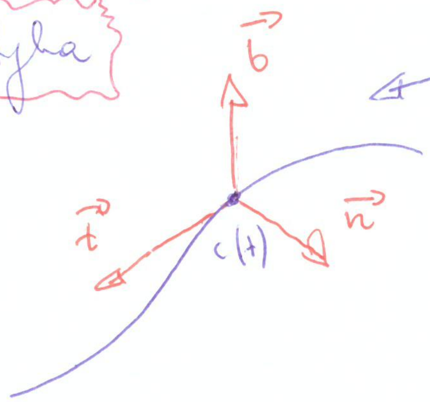
Podsumowanie wykład 2

Opis ruchu – kinematyka, dynamika.

Kinematyka



Trójścian Freneta

Krzywizna $\vec K(s) = \frac{d\vec t}{ds}$

Skręcenie $\vec T(s) = \frac{d\vec b}{ds}$

Równania Freneta-Serreta

R = macierz 3x3

$$\boxed{\frac{dR}{ds} = R \cdot A}$$

$$R = [\vec t, \vec n, \vec b] \qquad A = \begin{bmatrix} 0 & -K & 0 \\ K & 0 & -T \\ 0 & T & 0 \end{bmatrix}$$

Krzywizna i skręcenie determinują trajektorię

Dynamika

$$c(t) = \begin{pmatrix} c^1(t) \\ c^2(t) \\ \vdots \\ c^n(t) \end{pmatrix}$$ ← układ punktów materialnych

$c(t) \in R^{3n}$

Prawa ruchu – Zasady:

! – determinizm
– niezmienniczości (względności)

Prawo powszechnego ciążenia

Zasady dynamiki Newtona

$s(t)$

tor ruchu $c(s)$ a trajektoria ruchu $c(t)$

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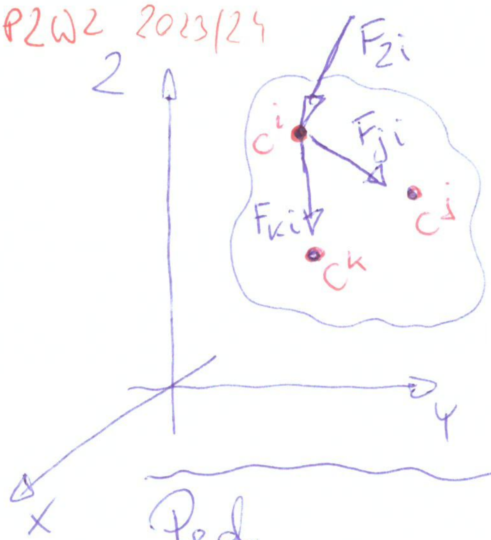

Dla każdego punktu c^i z II zasady dynamiki Newtona

$$\boxed{m_i \ddot{c}^i = F_i = F_{wi} + F_{zi} = \sum_{j \neq i} F_{ji} + F_{zi}}$$

Pęd

$$p_i = m_i \dot{c}^i \qquad P = \sum_{i=1}^{n} p_i$$

$$\dot{P} = \sum_{i=1}^{n} F_{zi} = 0$$

Zasada zachowania pędu

Moment pędu

$$M_i = c^i \times p_i \qquad M = \sum_{i=1}^{n} M_i$$

$$\dot{M} = \sum_{i=1}^{n} c^i \times F_{zi} = 0$$

Zasada zachowania momentu pędu

Energia

$$K_i = \frac{1}{2} m_i (\dot{c}^i, \dot{c}^i) \qquad K = \sum_{i=1}^{n} K_i$$

$$F_i = -\frac{\partial V}{\partial c^i} \rightarrow \qquad V$$

$$E = K + V$$

$$\dot{E} = \sum_{i=1}^{n} \left(\dot{c}^i, F_i + \frac{\partial V}{\partial c^i}\right) = 0$$

Zasada zachowania energii

Prawo ruchu $\rightarrow$ Trajektoria

$$\ddot{c} = F(c, \dot{c}) \qquad c(t) = \varphi(t, c(0), \dot{c}(0))$$

r.r. II rzędu — jego rozwiązanie

Przykłady

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① Funkcja potencjału $V(r) = -\frac{2}{3} r^{\frac{3}{2}}$

Prawo ruchu $\ddot{r} = r^{\frac{1}{2}}$ $\left(-\frac{\partial V}{\partial r}\right)$

Warunki początkowe $r(0) = \dot{r}(0) = 0$

Rozwiązanie trywialne $\boxed{r(t) = 0}$ I

Rozwiązanie „ogólne"

$$\begin{cases} \dot{r} = s \\ \dot{s} = r^{\frac{1}{2}} \end{cases} \Rightarrow \frac{ds}{dr} = \frac{r^{\frac{1}{2}}}{s}$$

$$s\,ds = r^{\frac{1}{2}} dr$$

$$\frac{1}{2} s^2 = \frac{2}{3} r^{\frac{3}{2}}$$

$$\boxed{s = \pm \frac{2}{\sqrt{3}} r^{\frac{3}{4}}}$$ ciągłe r.r.

$$r^{\frac{1}{4}} = \pm \frac{1}{2\sqrt{3}} t \Leftarrow \frac{dr}{dt} = \pm \frac{2}{\sqrt{3}} r^{\frac{3}{4}}$$

$$\Downarrow$$

$$\boxed{r(t) = \frac{1}{144} t^4}$$ II

Co z „determinizmem"?

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② Wahadło $ml^2\ddot{\varphi} = -mgl\sin\varphi \Rightarrow \ddot{\varphi} = -\sin\varphi$

$$\begin{cases} \varphi = q \\ \dot{\varphi} = p \end{cases}$$

$$\begin{cases} \dot{q} = p \\ \dot{p} = -\sin q \end{cases} \Rightarrow \frac{dp}{dq} = \frac{-\sin q}{p}$$

$$p\,dp = -\sin q\,dq$$

$$\boxed{p^2 = 2\cos q + C}$$ ☼

kiedy $f(q,\dot{q}) = C$? $\Leftarrow f(p,q) = p^2 - 2\cos q$

↑ orbity

lub rozwiązujemy ☼

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$f(x)$ $\quad R^n \to R^m$ $\quad \sqrt{2}$ $\quad \mathcal{X}$ - p. Banach

$f(x) = \sin x$ $\quad R \quad R^n$

R^n

$x, y \in \mathcal{X}$ $\quad \alpha, \beta \in R$

$f(x) \quad g(x)$

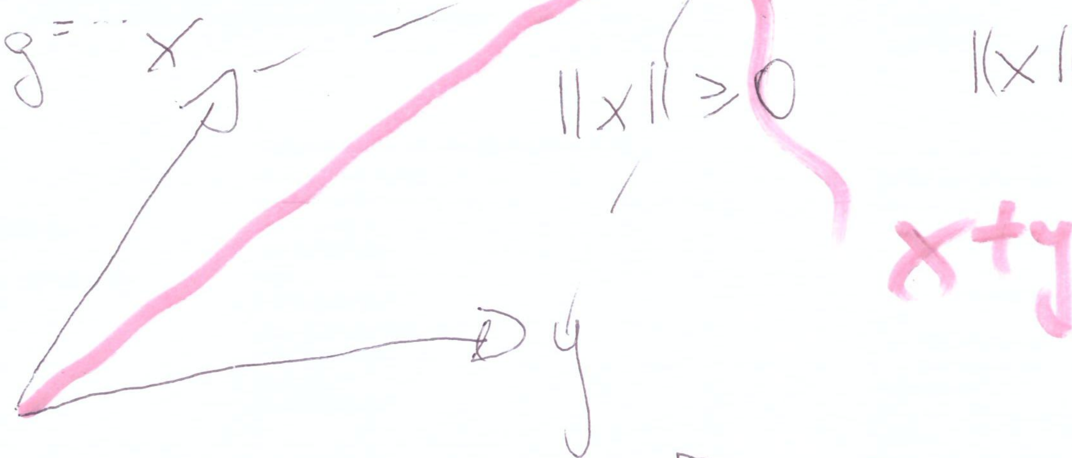
$\|\cdot\| : \mathcal{X} \to R_0^+$

$\alpha x + \beta y \in \mathcal{X}$

$\alpha f + \beta g = \dots$

$\|x\| \geq 0$

$\|x\| = 0 \iff x = 0$



$x \in R$

$\frac{1}{x}$ $\quad 1 \ \frac{1}{2} \ \frac{1}{3} \dots \lim \underline{0}$

$\sup(\sin(x)) = 1$ $\quad [0, 1]$

$\sup(0) = 0$ $\quad (0, 1)$

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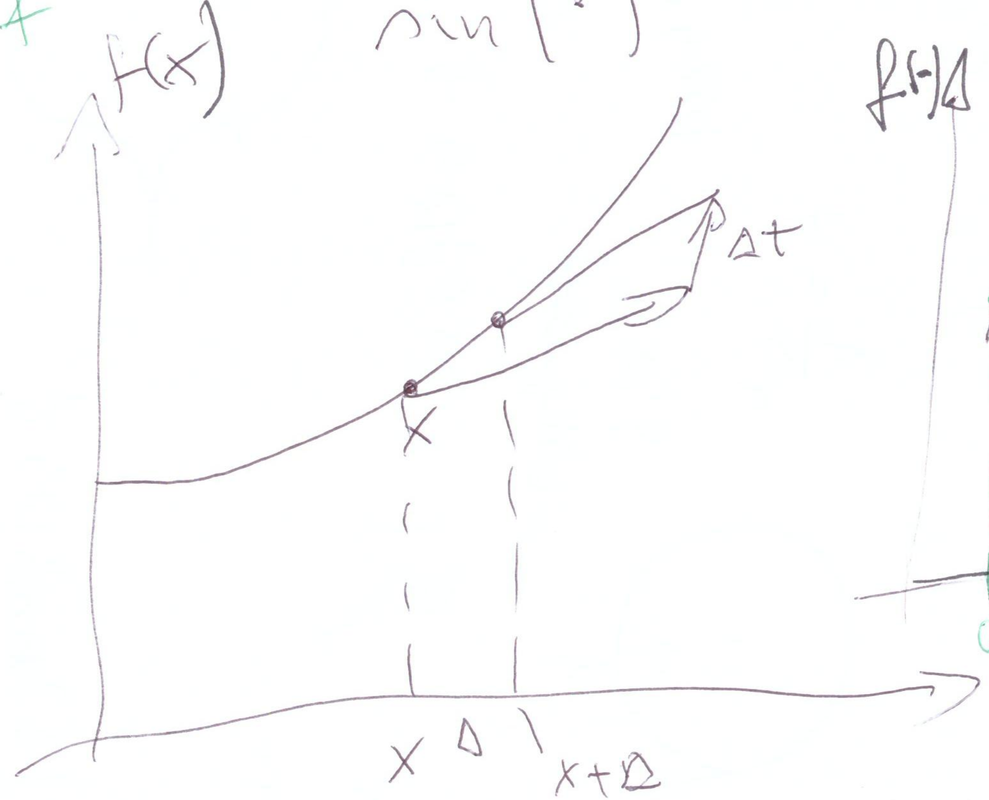
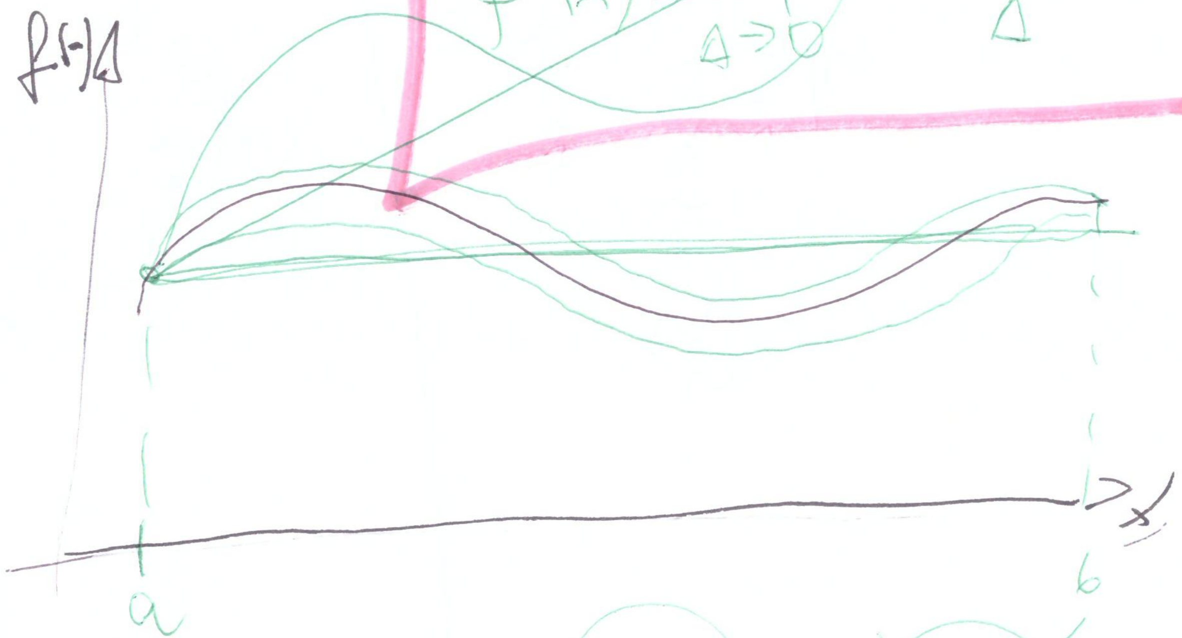
$f: R^n \to R^n$ $\sin(x)$ $\sin(1,2)$ $x \in R$

$f: R^n \to R$ $\sin\left(\frac{\pi}{4}\right)$

$\sin(\cdot)$



$$\boxed{f'(x) = \lim_{\Delta \to 0} \frac{f(x) - f(x+\Delta)}{\Delta}}$$



$f: \mathcal{X} \to \cancel{\mathcal{X}}$

R

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$$0 = Df = \lim_{v \to 0} \frac{\| f(x+v) - f(x) - Df_{x} v \|}{\| v \|}$$

$$f' \quad 0 = \lim_{\Delta \to 0} \frac{f(x) - f(x+\Delta) - f'\Delta}{\Delta}$$

$$\frac{d}{dt} \frac{1}{2} m_i (\dot c_i \dot c_i) = \cancel{\frac{1}{2}} m_i \left((\ddot c_i \dot c_i) + (\dot c_i \ddot c_i) \right)$$

$$\cancel{2}$$

$$K = \frac{1}{2} m v^2$$

$$\frac{1}{2} v^T m v$$

$$c_i \Rightarrow c^i$$

$$\frac{1}{2} m v^T v$$

$$(v, v)$$

$$V(c(t))$$

$$\frac{dV}{dt} = \frac{\partial V}{\partial c}^T \dot c \quad \left(\frac{\partial V}{\partial c} \dot c \right)$$

$$\boxed{F_i = -\frac{\partial V}{\partial c^i}}$$