Hill Climbing

Many search spaces are too big for systematic search.

A useful method in practice for some consistency and optimization problems is hill climbing:

- Assume a heuristic value for each assignment of values to all variables.
- Maintain an assignment of a value to each variable.
- Select a "neighbor" of the current assignment that improves the heuristic value to be the next current assignment.

Selecting Neighbors in Hill Climbing

- When the domains are small or unordered, the neighbors of a node correspond to choosing another value for one of the variables.

- When the domains are large and ordered, the neighbors of a node are the adjacent values for one of the dimensions.

- If the domains are continuous, you can use
  Gradient ascent: change each variable proportional to the gradient of the heuristic function in that direction. The value of variable X_i goes from v_i to $v_i + \eta \frac{\partial h}{\partial X_i}$.
  Gradient descent: go downhill; v_i becomes $v_i - \eta \frac{\partial h}{\partial X_i}$.

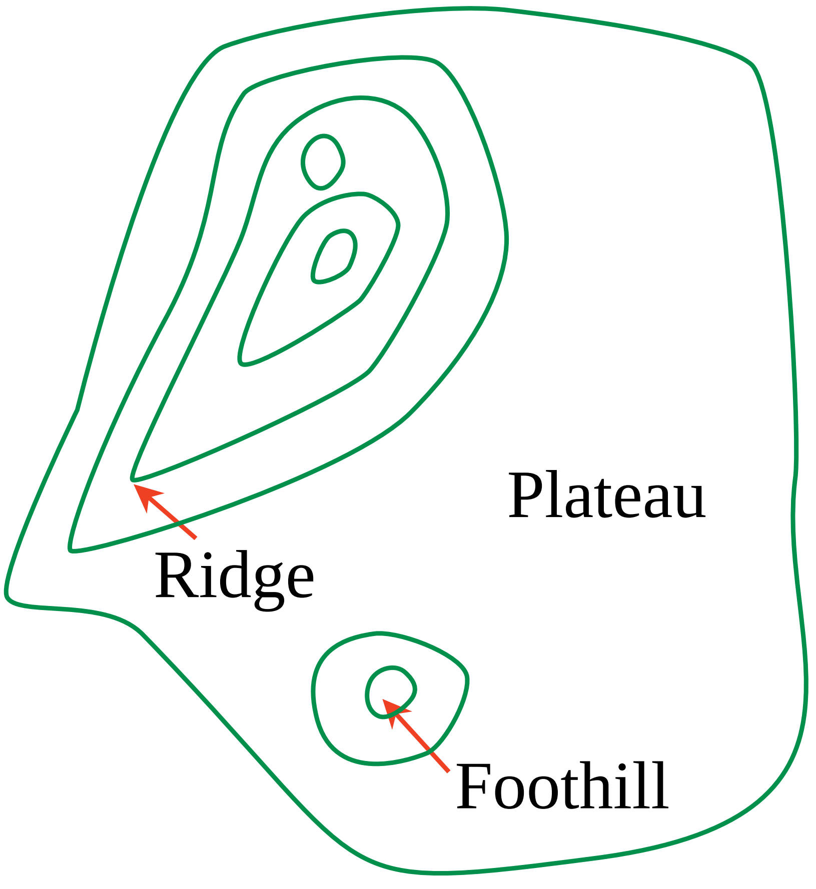
Problems with Hill Climbing

Foothills local maxima
that are not global
maxima

Plateaus heuristic values
are uninformative

Ridge foothill where
n-step lookahead
might help

Ignorance of the peak



Randomized Algorithms

- Consider two methods to find a maximum value:
  - Hill climbing, starting from some position, keep moving uphill & report maximum value found
  - Pick values at random & report maximum value found
- Which do you expect to work better to find a maximum?
- Can a mix work better?

Randomized Hill Climbing

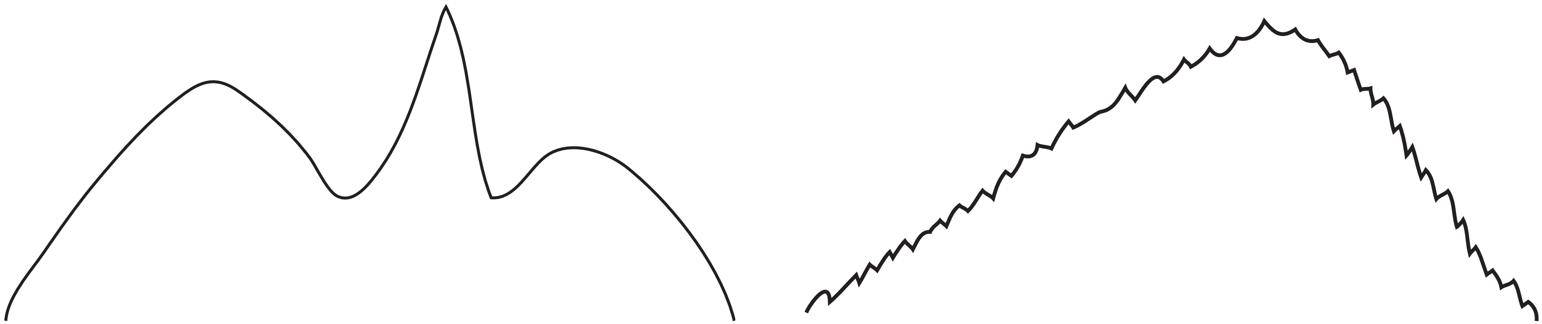
As well as uphill steps we can allow for:

- **Random steps:** move to a random neighbor.
- **Random restart:** reassign random values to all variables.

Which is more expensive computationally?

1-Dimensional Ordered Examples

Two 1-dimensional search spaces; step right or left:



- Which method would most easily find the maximum?
- What happens in hundreds or thousands of dimensions?
- What if different parts of the search space have different structure?

Stochastic Local Search for CSPs

- Goal is to find an assignment with zero unsatisfied relations.
- Heuristic function: the number of unsatisfied relations.
- We want an assignment with minimum heuristic value.
- Stochastic local search is a mix of:
  - Greedy descent: move to a lowest neighbor
  - Random walk: taking some random steps
  - Random restart: reassigning values to all variables

Greedy Descent

- It may be too expensive to find the variable-value pair that minimizes the heuristic function at every step.
- An alternative is:
  - Select a variable that participates in the most number of conflicts.
  - Choose a (different) value for that variable that resolves the most conflicts.

The alternative is easier to compute even if it doesn't always maximally reduce the number of conflicts.

Random Walk

You can add randomness:

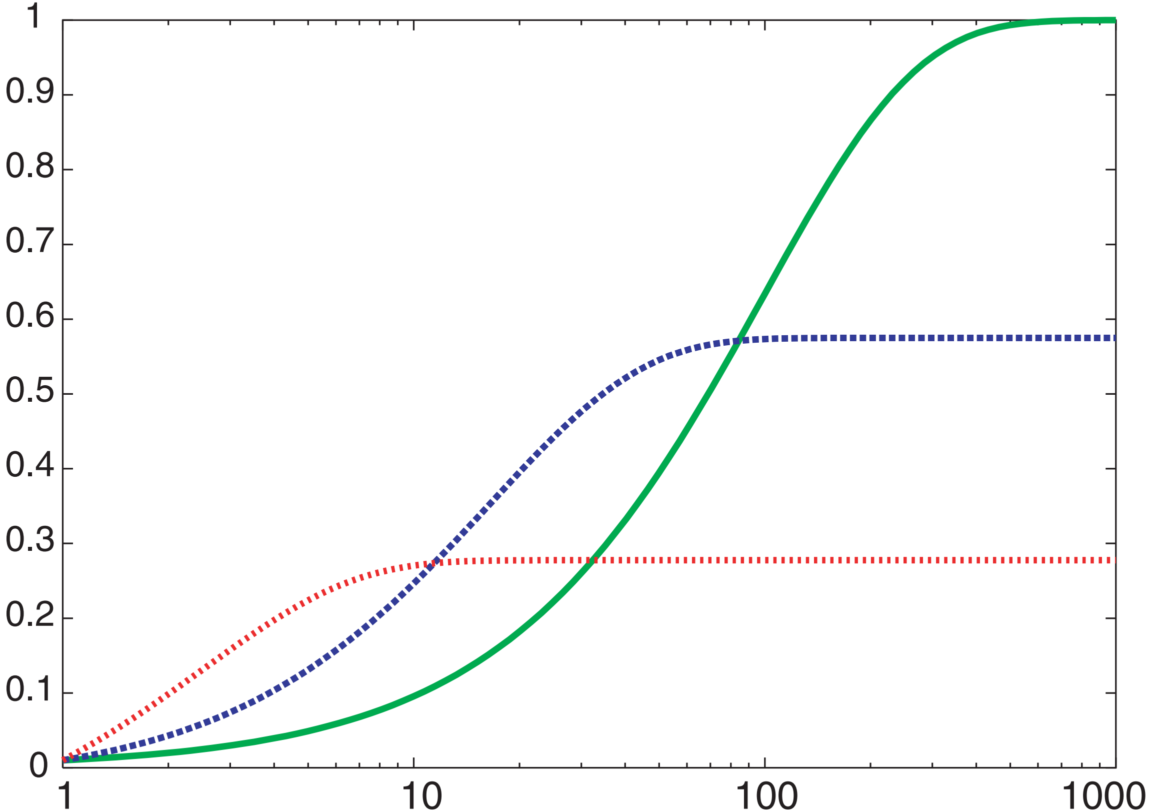
- When choosing the best variable-value pair, randomly sometimes choose a random variable-value pair.
- When selecting a variable then a value:
  - Sometimes choose a random variable.
  - Sometimes choose, at random, a variable that participates in a conflict (a red node).
  - Sometimes choose a random variable.
- Sometimes choose the best value and sometimes choose a random value.

Comparing Stochastic Algorithms

- How can you compare three algorithms when
  - one solves the problem 30% of the time very quickly but doesn't halt for the other 70% of the cases
  - one solves 60% of the cases reasonably quickly but doesn't solve the rest
  - one solves the problem in 100% of the cases, but slowly?
- Summary statistics, such as mean run time, median run time, and mode run time don't make much sense.

Runtime Distribution

- Plots runtime (or number of steps) and the proportion (or number) of the runs that are solved within that runtime.



Variant: Simulated Annealing

- Pick a variable at random and a new value at random.
- If it is an improvement, adopt it.
- If it isn't an improvement, adopt it probabilistically depending on a temperature parameter, T.
  - With current node n and proposed node n' we move to n' with probability $e^{(h(n')-h(n))/T}$
- Temperature can be reduced.

Tabu lists

- To prevent cycling we can maintain a tabu list of the k last nodes visited.
- Don't allow a node that is already on the tabu list.
- If $k = 1$, we don't allow a node to the same value.
- We can implement it more efficiently than as a list of complete nodes.
- It can be expensive if k is large.

Parallel Search

- Idea: maintain k nodes instead of one.
- At every stage, update each node.
- Whenever one node is a solution, it can be reported.
- Like k restarts, but uses k times the minimum number of steps.

Beam Search

- Like parallel search, with k nodes, but you choose the k best out of all of the neighbors.
- When $k = 1$, it is hill climbing.
- When $k = \infty$, it is breadth-first search.
- The value of k lets us limit space and parallelism.
- Randomness can also be added.

Stochastic Beam Search

- Like beam search, but you probabilistically choose the k nodes at the next generation.
- The probability that a neighbor is chosen is proportional to the heuristic value.
- This maintains diversity amongst the nodes.
- The heuristic value reflects the fitness of the node.
- Like asexual reproduction: each node gives its mutations and the fittest ones survive.

Genetic Algorithms

- Like stochastic beam search, but pairs are nodes are combined to create the offspring:
- For each generation:
  - Randomly choose pairs of nodes where the fittest individuals are more likely to be chosen.
  - For each pair, perform a cross-over: form two offspring each taking different parts of their parents:
  - Mutate some values
- Report best node found.

Crossover

- Given two nodes:

$$X_1 = a_1, X_2 = a_2, \ldots, X_m = a_m$$

$$X_1 = b_1, X_2 = b_2, \ldots, X_m = b_m$$

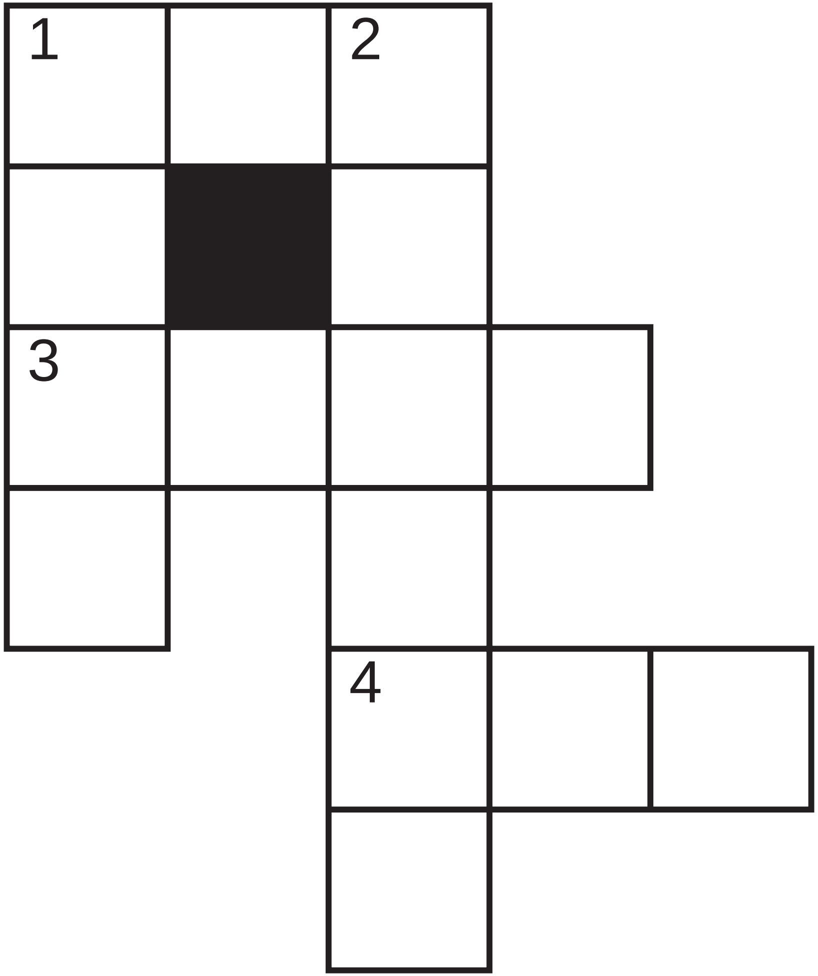
- Select i at random.
- Form two offspring:

$$X_1 = a_1, \ldots, X_i = a_i, X_{i+1} = b_{i+1}, \ldots, X_m = b_m$$

$$X_1 = b_1, \ldots, X_i = b_i, X_{i+1} = a_{i+1}, \ldots, X_m = a_m$$

- Note that this depends on an ordering of the variables.
- Many variations are possible.

Example: Crossword Puzzle



Words:

ant, big, bus, car, has

book, buys, hold,

lane, year

beast, ginger, search,

symbol, syntax