Scientific & Engineering Programming

II Year Electronics and Computer Engineering, FoEPhaM, WUST

7 Mathematica Lab Class 7 – Repetere

7.1 The scope

To review the basic principles of Mathematica.

7.2 Prerequisites

Before the classes you should know, how to solve previous task lists.

7.3 Tasks

1. For matrices

$$R_1 = \begin{bmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} & 0 \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} & 0 \\ 0 & 0 & 1 \end{bmatrix}, R_2 = \begin{bmatrix} \cos\left(-\frac{\pi}{4}\right) & -\sin\left(-\frac{\pi}{4}\right) & 0 \\ \sin\left(-\frac{\pi}{4}\right) & \cos\left(-\frac{\pi}{4}\right) & 0 \\ 0 & 0 & 1 \end{bmatrix}, I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

   verify that

   (a) $R_1 \cdot R_2 - I = 0$

   (b) $R_1^T = R_1^{-1}$

   (c) $\det R_1 = \det R_2 = \det(R_1 \cdot R_2) = 1$

2. Let $p_1(s), p_2(s), p_3(s)$ be polynomials of the variable s with real coefficients such that

$$p_1(s) = s^3 + 3s + 4,\ p_2(s) = s^2 + 1,\ p_3(s) = p_1(s)p_2(s).$$

   Compute the coefficients of $p_3(s)$. Determine the roots of $p_s(s)$ and next verify the correctness of the result.

3. Define the function $sum(x)$ which for its argument being a list returns the sum of list elements.

4. Define the function $sumPositive(x)$ which for its argument being a list returns the sum of list positive elements.

5. Define the function $poly(c)$ which for its argument being a vector $(c_0, c_1, c_2, c_3, \ldots)$ returns the polynomial of x with the vector elements taken as the polynomial coefficients: $c_0 + c_1x + c_2x^2 + c_3x^3 + \ldots$

6. Let $f(x) = (x + a)^2(x - b)^3$ where $a, b \in \mathbb{R}$. Find the extremes of this function.

7. Let $f(x) = \frac{x}{1+x^4}$. Plot $f(x)$, $\frac{df(x)}{dx}$, and $\int_o^x f(x)dx$ for $x \in [0, 10]$.

8. Consider a function $f : \mathbb{R}^2 \to \mathbb{R}$ such that $f(x, y) = xy$ and a curve K given in a parametric form
$$K = \left\{ (x, y)^T \in \mathbb{R} \;\middle|\; x(t) = a\cos(t), y(t) = b\sin(t), 0 \le t \le \frac{\pi}{2} \right\}.$$
Plot $f(x, y)$ for $0 \le x \le 2$ and $0 \le y \le 2$. Plot K with $a = 2$, and $b = 1$. Compute $\int_K f(x, y)dt$, which, as it follows from calculus, can be expressed as
$$\int_{t_{\min}}^{t_{\max}} f(x(t), y(t))\sqrt{x'(t)^2 + y'(t)^2}dt$$

9. Solve numerically (and optionally symbolically) the set of equations
$$\begin{cases} \frac{dx_1}{dt} = x_2 \\ \frac{dx_2}{dt} = -x_1 - kx_2 \end{cases}.$$
Visualize the result with time plot and phase portrait (state space plot) and interpret the results for different initial conditions and values of the parameter k (large: $k \ge 2$ and small: $0 \le k < 2$).

10. Solve numerically the set of equations
$$\begin{cases} \frac{dx}{dt} = \sigma(y - x) \\ \frac{dy}{dt} = x(\rho - z) - y \\ \frac{dz}{dt} = xy - \beta z \end{cases}.$$
Visualize the results (time plot, state space plot, xz plane) for different system parameters (start with $\sigma = 10$, $\beta = 8/3$, $\rho = 28$). Examine the influence of initial conditions to the system trajectory (one may start with $(0, 1, 0)$, what Lorenz considered :).