Scientific & Engineering Programming

II Year Electronics and Computer Engineering, FoEPhaM, WUST

5 Mathematica Lab Class 5 – Dynamical systems

5.1 The scope

To get familiar with the methodology of dynamical systems simulations in Mathematica, methods for results visualization and analysis.

5.2 Prerequisites

Before the classes you should know, how to:

- represent and define differential equations,
- solve numerically differential equations,
- visualize functions being the differential equations solutions,
- model simple physical systems.

5.3 Tasks

5.3.1 Simple oscillators

1. Recall the harmonic oscillator from the task 8, Mathematica Lab Class 4. Visualize it with a simple animation of the system movement (e.g. as a mass on a spring or a single pendulum).

2. Equip the harmonic oscillator with a damper, which generates the friction force proportional to the movement velocity $F_f = -c\frac{dx}{dt}$, where c is called the viscous damping coefficient. Repeat the simulations. Analyze the system behavior for different values of the system damping ratio $\zeta = \frac{c}{2\sqrt{mk}}$: ζ larger than, equal to, or smaller than 1 (base on time plots of oscillator position, animation from Point 1.)

3. Consider a driven harmonic oscillator with a damper, affected by an externally applied force $F(t)$.

   - Analyze the step response of the system.
   - Apply a sinusoidal driving force $F(t) = F_0 \sin(\omega t)$, where F_0 is the driving amplitude and ω is the driving frequency. Analyze the system behavior for different values of the driving frequency ω (ω larger than, equal to, or smaller than $\omega_0 = \sqrt{\frac{k}{m}}$.)

5.3.2 Double pendulum

4. For the double pendulum model derived during the lecture with the first approach (the file with the pendulum model is provided on the course web page in the laboratory classes table – use it as a start point) repeat the simulations shown during the lecture. Visualize the obtained results (time plots, the animation). Run the simulation with different initial conditions. Repeat the simulations for different system parameters.

5. Extend the double pendulum model with an additional link. Derive the model and perform the simulations.