Scientific & Engineering Programming

II Year Electronics and Computer Engineering, FoEPhaM, WUST

2 Mathematica Lab Class 2 – Mathematica basics 2

2.1 The scope

To get familiar with tools, work methodology, and Mathematica interfaces. To perform basic calculations with vectors, matrices, and loops.

2.2 Prerequisites

Before the classes you should know, how to:

- define vectors and matrices manually,
- generate vectors and matrices with use of Mathematica functions (`Table`, `Do`, `For`)
- multiply vectors by scalars and vectors,
- multiply matrices by scalars, vectors, and matrices,
- transpose matrices, compute their determinants, inverses.

2.3 Tasks

2.3.1 Vectors and matrices basic operations

1. For the matrices A and B compute (if possible): $3A - \frac{1}{2}B$, A^T, AB, BA, A^2:
   (a) $A = \begin{bmatrix} 1 & 4 \\ -2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -6 \\ -8 & 2 \end{bmatrix}$,
   (b) $A = \begin{bmatrix} 1 & -3 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -4 & 0 \end{bmatrix}$,
   (c) $A = \begin{bmatrix} 1 \\ 0 \\ 3 \\ 0 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 1 & 0 & 5 \end{bmatrix}$,
   (d) $A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & -4 \\ -3 & 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 0 \\ 4 & 1 \\ 0 & 3 \end{bmatrix}$.
   (e) What will change in the computation in points (b) and (c), if one represents the objects A and B as vectors?

2. Find the determinants of matrices:
   (a) $\begin{bmatrix} 5 & -1 \\ 7 & -8 \end{bmatrix}$,
   (b) $\begin{bmatrix} \cos\frac{\pi}{12} & \sin\frac{\pi}{12} \\ \sin\frac{\pi}{12} & \cos\frac{\pi}{12} \end{bmatrix}$,
   (c) $\begin{bmatrix} 2 & 5 & 3 \\ 4 & 0 & 8 \\ 7 & 1 & 1 \end{bmatrix}$,
   (d) $\begin{bmatrix} 1 & 66 & 777 \\ 2 & 55 & 888 \\ 3 & 44 & 999 \end{bmatrix}$.

3. For the matrices from the task 1 find their inverses when possible.

2.3.2 Vectors and matrices generation

4. Generate the following vectors:
   (a) a vector $v \in R^5$, with components increasing by 5 and starting with 11,
   (b) 5 vectors $v_i \in R^5$, $i \in [0, 4]$, with components increasing by 2 and starting with i,
   (c) 5 vectors $v_i \in R^5$, $i \in [0, 4]$, with subsequent components squared and starting with i,
   (d) versors for R^5,
   (e) normalized versions of the above,
   (f) vectors with reversed elements order of the vectors from points 4b, and 4c,
   (g) vectors with exchanged elements 2 and 4 of the vectors from points 4b, and 4c.
5. Generate the following matrices:
   (a) null matrix of size 5x5,
   (b) unit matrix of size 5x5,
   (c) diagonal matrix of size 5x5 with elements $a_{ii} = i$,
   (d) diagonal matrix of size 5x5 with elements $a_{ii} = i^i$,
   (e) the above matrix with element a_{22} set to 7,
   (f) the above matrix with exchanged rows 2 and 3,
   (g) the above matrix with exchanged columns 3 and 5,
   (h) the above matrix with the second row zeroed,
   (i) the above matrix with zeroed the upper left submatrix of size 2x2.
6. Find the determinants of square matrices $A = \left[a_{ij}\right]$ of size 5x5 ($1 \leq i, j \leq 5$), if:
   (a) $a_{ij} = i + j$,
   (b) $a_{ij} = 2i + 3j$,
   (c) $a_{ij} = i \cdot j$,
   (d) $a_{ij} = i^2 j^3$,
   (e) $a_{ij} = \min(i, j)$,
   (f) $a_{ij} = i^j$.

2.3.3 Tasks with content

7. Calculate the surface area of a parallelogram constituted with two vectors $u = \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix}$, $v = \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix}$.
8. Let $a = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $b = \begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix}$, and $c = \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}$ be the vectors defining a three-dimensional parallelepiped. Find its volume.