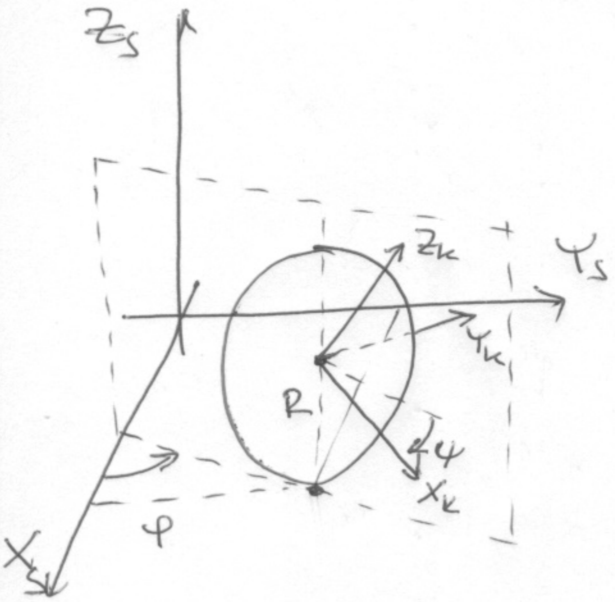
Mechanika analityczna, II r AiR

Wykład 13

Dynamika ciała sztywnego – przykłady

1. Koło pionowe



$m, I_1 = I_3, I_2, I_{B_1}, R$

Kąty Eulera: $R(Z,\varphi)R(Y,\theta)R(Z,\psi)$

$\varphi=\varphi,\ \theta=\psi,\ \psi=0$

$$L = \tfrac{1}{2}\left(I_{B_1} s_\theta^2 c_\psi^2 + I_{B_2} s_\theta^2 s_\psi^2 + I_{B_3} c_\theta^2\right)\dot\varphi^2 + \tfrac{1}{2}\left(I_{B_1} s_\psi^2 + I_{B_2} c_\psi^2\right)\dot\theta^2 + \tfrac{1}{2} I_{B_3}\dot\psi^2$$
$$+ \left(-I_{B_1} s_\theta s_\psi c_\psi + I_{B_2} s_\theta s_\psi c_\psi\right)\dot\varphi\dot\theta + I_{B_3} c_\theta \dot\varphi\dot\psi + \tfrac{1}{2} m_B\left(\dot T_1^2 + \dot T_2^2 + \dot T_3^2\right)$$
$$- m_B g T_3$$

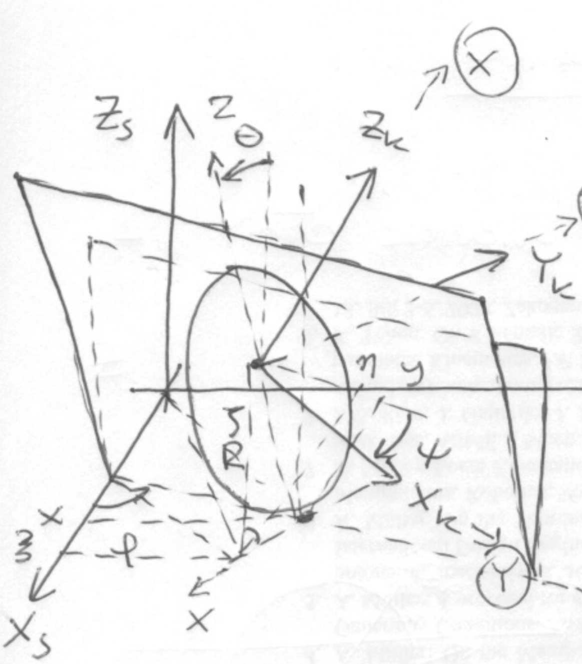
Podstawiamy: $I_{B_1} = I_{B_3} = I_1,\ I_{B_2} = I_2,\ m_B = m,\ T_1 = x,\ T_2 = y,\ T_3 = R$

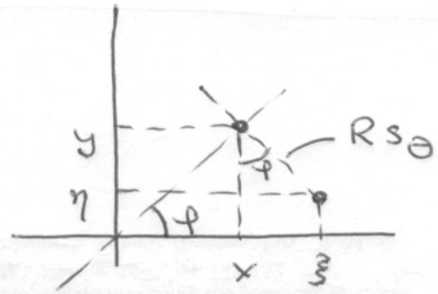
Otrzymujemy

$$L = \tfrac{1}{2}\left(I_1 s_\psi^2 + I_1 c_\psi^2\right)\dot\varphi^2 + \tfrac{1}{2} I_2 \dot\psi^2 + \tfrac{1}{2} m\left(\dot x^2 + \dot y^2\right) - mgR =$$
$$\cong \tfrac{1}{2} I_1 \dot\varphi^2 + \tfrac{1}{2} I_2 \dot\psi^2 + \tfrac{1}{2} m\left(\dot x^2 + \dot y^2\right)$$

2. Koło pochylone

$$m, I_1 = I_3, I_2, R$$





$$T_1 = \xi = R s_\theta s_\varphi + x, \quad T_2 = y - R s_\theta c_\varphi$$

$$T_3 = \zeta = R c_\theta$$

Kąty Eulera: $R(Z,\varphi)R(Y,\theta)R(Z,\psi)$

$$\varphi = \varphi - 90^\circ, \quad \theta = \theta - 90^\circ, \quad \psi = \psi$$

$$I_{B_1} = I_{B_2} = I_1, \quad I_{B_3} = I_2$$

$$L = \frac{1}{2}(I_1 s_\theta^2 c_\psi^2 + I_1 c_\theta^2 s_\psi^2 + I_2 s_\theta^2)\dot{\varphi}^2 + \frac{1}{2}(I_1 s_\psi^2 + I_1 c_\psi^2)\dot{\theta}^2 + \frac{1}{2} I_2 \dot{\psi}^2 +$$

$$+ I_2 s_\theta \dot{\varphi}\dot{\psi} + \frac{1}{2} m(\dot{T}_1^2 + \dot{T}_2^2 + \dot{T}_3^2) - V = K - V$$

$$K_R = \frac{1}{2} I_1 c_\theta^2 \dot{\varphi}^2 + \frac{1}{2} I_2 s_\theta^2 \dot{\varphi}^2 + \frac{1}{2} I_1 \dot{\theta}^2 + \frac{1}{2} I_2 \dot{\psi}^2 + I_2 s_\theta \dot{\varphi}\dot{\psi}$$

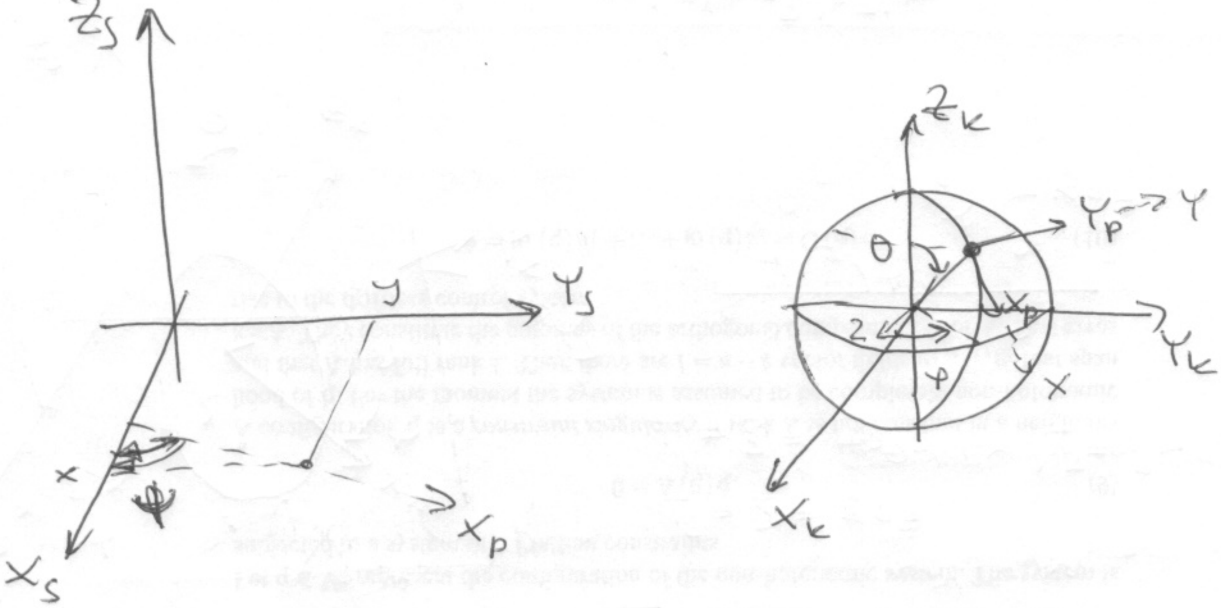
Jest to ten sam wynik, który uzyskaliśmy poprzednio

$$K_R = \frac{1}{2} I_1 (\dot{\theta}^2 + \dot{\varphi}^2 c_\theta^2) + \frac{1}{2} I_2 (\dot{\psi} + \dot{\varphi} s_\theta)^2 = \frac{1}{2} I_1 \dot{\theta}^2 + \frac{1}{2} I_1 \dot{\varphi}^2 c_\theta^2 +$$

$$+ \frac{1}{2} I_2 \dot{\psi}^2 + \frac{1}{2} I_2 \dot{\varphi}^2 s_\theta^2 + I_2 s_\theta \dot{\varphi}\dot{\psi}$$

3. Kula tocząca się

$$m, I, R$$



$$T_1 = x,\ T_2 = y,\ T_3 = R$$

Kąty Eulera: $R(Z,\varphi)R(Y,\theta)R(Z,\psi)$

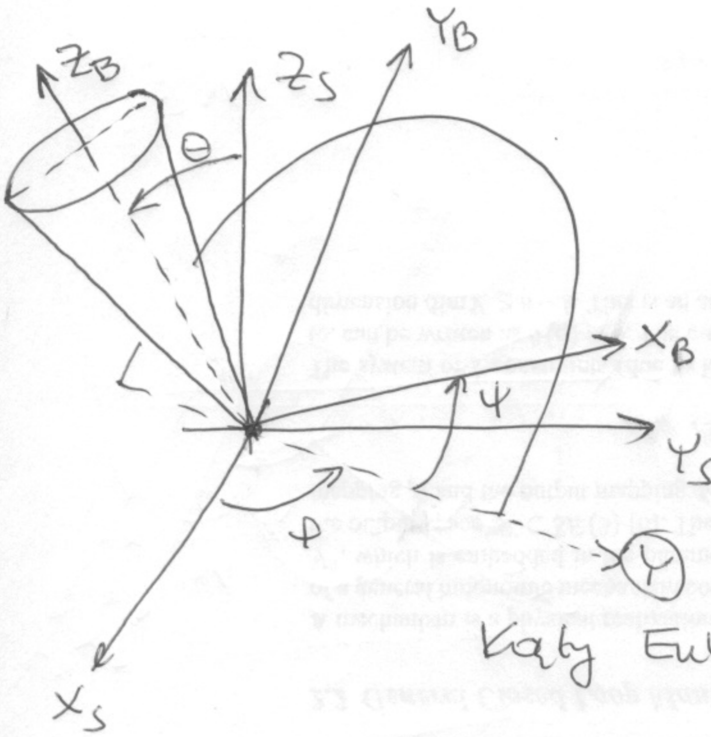
$$\varphi = -\psi,\ \theta = -\theta,\ \psi = -\varphi$$

$$K_R = \frac{1}{2} I \left(s_\theta^2 c_\varphi^2 + s_\theta^2 s_\varphi^2 + c_\theta^2\right)\dot\psi^2 + \frac{1}{2} I \dot\theta^2 + \frac{1}{2} I \dot\varphi^2 + I \dot\varphi\dot\psi c_\theta$$

$$K_R = \frac{1}{2} I\left(\dot\varphi^2 + \dot\theta^2 + \dot\psi^2\right) + I\dot\varphi\dot\psi c_\theta$$

Wynik jest zgodny z uzyskanym poprzednio.

4. Bąk Lagrange'a



$$T=0,\ \dot{T}=0$$

$$I_{B_1}=I_{B_2}=I_1,\ I_{B_3}=I_2$$

Kąty Eulera: $R(Z,\varphi)R(Y,\theta)R(Z,\psi)$

$$\varphi=\varphi-90^\circ,\ \theta=\theta,\ \psi=\psi+90^\circ$$

$$K=\frac{1}{2}\left(I_1 s_\theta^2 s_\psi^2+I_1 s_\theta^2 c_\psi^2+I_2 c_\theta^2\right)\dot{\varphi}^2+\frac{1}{2}I_1\dot{\theta}^2+\frac{1}{2}I_2\dot{\psi}^2+I_2\dot{\varphi}\dot{\psi}c_\theta$$

$$K=\frac{1}{2}I_1 s_\theta^2\dot{\varphi}^2+\frac{1}{2}I_2 c_\theta^2\dot{\varphi}^2+\frac{1}{2}I_1\dot{\theta}^2+\frac{1}{2}I_2\dot{\psi}^2+I_2\dot{\varphi}\dot{\psi}c_\theta$$

Wynik zgodny z otrzymanym wcześniej:

$$K=\frac{1}{2}I_1\left(\dot{\theta}^2+\dot{\varphi}^2 s_\theta^2\right)+\frac{1}{2}I_2\left(\dot{\psi}+\dot{\varphi}c_\theta\right)^2=\frac{1}{2}I_1\dot{\theta}^2+\frac{1}{2}I_1 s_\theta^2\dot{\varphi}^2+$$

$$+\frac{1}{2}I_2\dot{\psi}^2+\frac{1}{2}I_2 c_\theta^2\dot{\varphi}^2+I_2\dot{\varphi}\dot{\psi}c_\theta$$